General

Existence:
Consider

$$Lu = -u_x - iu_y + 2i(x+iy)u_z$$

Theorem: There exists a function $f \in C^\infty(R^3)$. Such that the equation $Lu = f$ has now solutions in $C^1(\Omega)$ and $\Omega \subset R^3$.

Fourier transform:

$$\begin{aligned}
\widehat{u}(k,t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ikx}u(x,t)dx \\
u(x,t) &= \int_{-\infty}^{\infty} e^{+ikx}\widehat{u}(k,t)dk
\end{aligned}$$

Wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$$

Take Fourier transform of both sides

$$\begin{aligned}
\frac{\partial^2 \widehat{u}}{\partial t^2} &= -c^2k^2\widehat{u} \\
\widehat{u}(k,t) &= A\cos(ckt) + B\sin(ckt)
\end{aligned}$$

i.e. oscillatory solution

Heat equation

$$\frac{\partial u}{\partial t} = \alpha\frac{\partial^2 u}{\partial x^2}$$

Take Fourier transform of both sides

$$\begin{aligned}
\frac{\partial \widehat{u}}{\partial t} &= -\alpha k^2\widehat{u} \\
\widehat{u}(k,t) &= Ae^{-\alpha k^2 t}
\end{aligned}$$

i.e. decays if $\alpha > 0$ and grows (rapidly for high frequencies) if $\alpha < 0$.
In fact the equation is ill-posed if $\alpha < 0$.