Karhunen-Loeve (KL) transformation

Autocovariance:

$$C(i,j)=\frac{1}{N^2}\sum_{k=1}^{N}\sum_{l=1}^{N}\left(x_i(k,l)-x_{i0}\right)\left(x_j(k,l)-x_{j0}\right)$$

where $x_i(k,l)$ is the "gray" level of pixel (k,l) at band i (i=1...3 for RGB)

x_{i0} is the mean of band i

$E\{\}$ is the expected value over all outcomes of the random experiment. So

$$C(i,j)=E\left\{\left(x_i(k,l)-x_{i0}\right)\left(x_j(k,l)-x_{j0}\right)\right\}$$

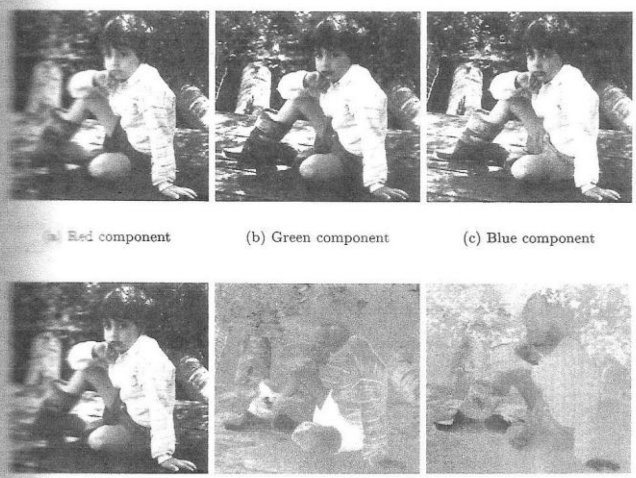
and $C(i,j)$ is a 3×3 matrix.

If the data is uncorrelated then C is diagonal. So we wish to diagonalize C.

Procedure

1. Find the mean of the distribution of points in color space (R_0,G_0,B_0)
2. Subtract the mean from the gray level in each corresponding band.
3. Find the autocorrelation matrix $C(i,j)$ of the original image
4. Find the eigenvalues of $C(i,j)$ and arrange them in **decreasing** order. Form the matrix A where the rows of A are the eigenvectors of C
5. Let $x=\begin{pmatrix}R\\G\\B\end{pmatrix}$ and define $y=Ax$

6. The new colors are linear combinations of the intensity values of the original colors arranged so that the first component has the most information for the image.
7. The information in each band is maximal for the number of given bits
8. To convert to a monochromatic image use only the first component and this gives the maximal information.
9. Note: the principle components DO NOT correspond to any physical color



(a) Red component | (b) Green component | (c) Blue component

(d) First principal component | (e) Second principal component | (f) Third principal component

Figure 4.8: Example of principal component analysis of a colour image.

Example 4.2

Is it possible for matrix C below to represent the autocovariance matrix of a three-band image?

$$C = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ -2 & 2 & 0 \end{pmatrix}$$

This matrix cannot represent the autocovariance matrix of an image because from equation (4.10) it is obvious that C must be symmetric with positive elements along its diagonal.