Total Variation Blind Deconvolution

Outline

- Blind Deconvolution formulation
- Numerical methods solving PDE’s
- Choice of regularization parameters
- Results
- Summary

Image Restoration With Known PSF

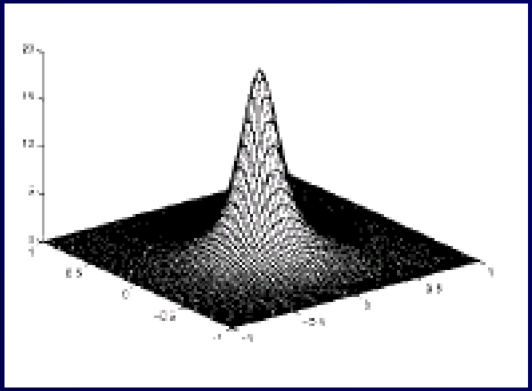
Blurring model: $z = h * u + \eta$

$$\min_u f(u) = \min_u \left\{ \underbrace{\frac{1}{2}\left\| h * u - z \right\|_{L^2(\Omega)}^2}_{\text{Denoising}} + \underbrace{\alpha \int_\Omega \left| \nabla u \right| dxdy}_{\text{TV smoothing}} \right\}$$

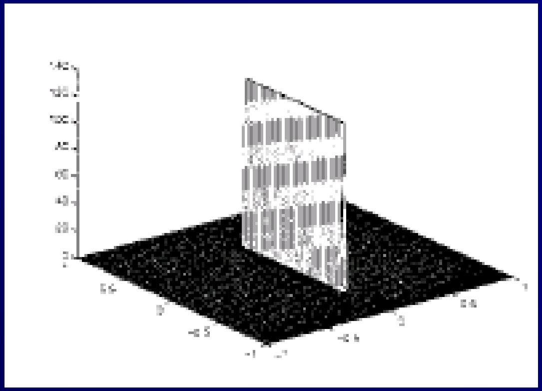
Rudin, Osher, Fetami 1992

Blind Deconvolution

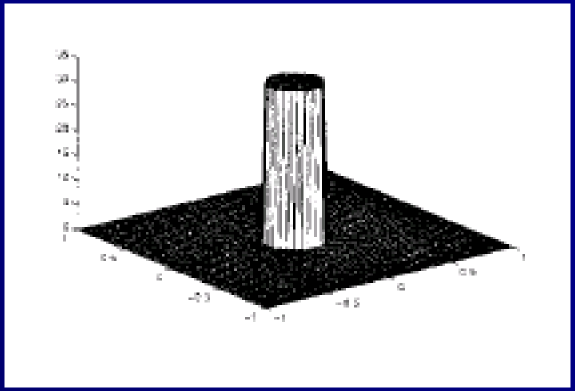
- Space invariant image restoration (of u and h) without any a priory knowledge of the PSF
- Assuming PSF is piecewise smooth



Atmospheric turbulence



Motion blur



Out of focus

Blind Deconvolution Formulation

$$\min_{u,h} f(u,h) = \min_{u,h} \frac{1}{2}\left\| h * u - z \right\|^2_{L^2(\Omega)} +$$

$$\alpha_1 \int_\Omega \left|\nabla u\right| dxdy + \alpha_2 \int_\Omega \left|\nabla h\right| dxdy$$

Euler-Lagrange optimization yields:

$$\frac{\delta L}{\delta h} = \left(u * h - z\right) * u(-x,-y) - \alpha_2 \nabla \cdot \left(\frac{\nabla h}{\left|\nabla h\right|} \right) = 0$$

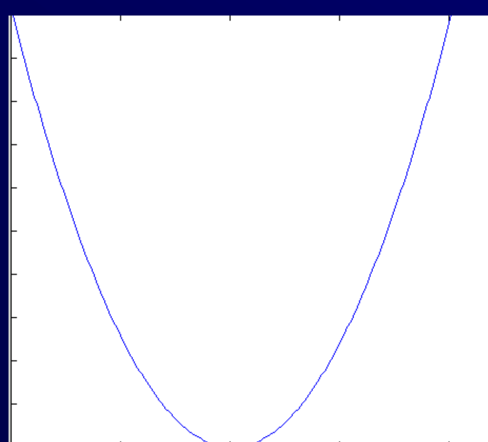
$$\frac{\delta L}{\delta u} = \left(u * h - z\right) * h(-x,-y) - \alpha_1 \nabla \cdot \left(\frac{\nabla u}{\left|\nabla u\right|} \right) = 0$$

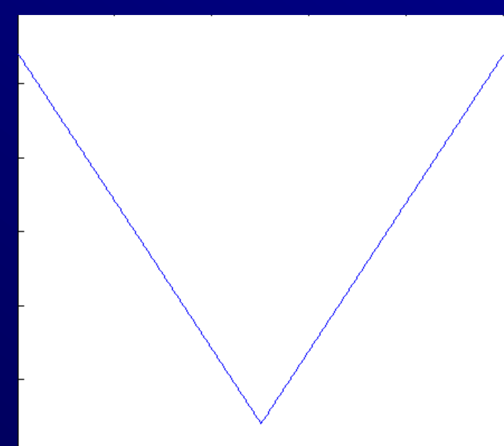
Convexity

Does the problem have global or local minima?

$$f(u,h)=\iint_{\Omega}(h*u-z)^2+\alpha_1|\nabla u|+\alpha_2|\nabla h|\,dxdy$$

$f(u,h)$ is not jointly convex, but for a given u, $f(u,\cdot)$ is convex with respect to h and vise versa.





$$\min_{u,h} f(u,h) = \min_{u,h} \frac{1}{2}\left\| h*u - z \right\|_{L^2(\Omega)}^2 +$$

$$\alpha_1 \int_\Omega |\nabla u| dxdy + \alpha_2 \int_\Omega |\nabla h| dxdy$$

Minimization problem may not have a unique solution [e.g. (-u,-h) is also solution], therefore the following conditions are imposed:

$$\int_\Omega h(x,y)dxdy = 1$$

$$u(x,y), h(x,y) \geq 0$$

$$h(x,y) = h(-x,-y)$$

Numerical Methods

AM – Alternate Minimization

$$Initiallization: u^0 = z, h^0 = \delta(x, y)$$

$$f(u^0, h^1) = \min_h f(u^0, \cdot)$$

$$f(u^1, h^1) = \min_u f(\cdot, h^1)$$

The convexity property yields that the function $f(u^n, h^n)$ always decreases when n increases

$$Solve\ h^{n+1} : u^n(-x,-y) * (u^n * h^{n+1} - z) - \alpha_2 \nabla \cdot \left(\frac{\nabla h^{n+1}}{\left|\nabla h^{n+1}\right|} \right) = 0$$

$$Solve\ u^{n+1} : h^{n+1}(-x,-y) * (h^{n+1} * u^{n+1} - z) - \alpha_1 \nabla \cdot \left(\frac{\nabla u^{n+1}}{\left|\nabla u^{n+1}\right|} \right) = 0$$

Linearization of PDE's

Lagged Diffusitivity Fixed Point (FP) ,Vogel and Oman 1996

Coefficients are lagged by one iteration and then linear equations are solved:

$$Solve\ h^{n+1} : u^n(-x,-y) * (u^n * h_{i+1}^{n+1} - z) - \alpha_2 \nabla \cdot \left(\frac{\nabla h_{i+1}^{n+1}}{\left| \nabla h_i^{n+1} \right|} \right) = 0$$

$$Solve\ u^{n+1} : h^{n+1}(-x,-y) * (h^{n+1} * u_{i+1}^{n+1} - z) - \alpha_1 \nabla \cdot \left(\frac{\nabla u_{i+1}^{n+1}}{\left| \nabla u_i^{n+1} \right|} \right) = 0$$

Impose:

$$h^{n+1}(x,y) = \begin{cases} h^{n+1}(x,y), & h^{n+1}(x,y) > 0 \\ 0 & otherwise \end{cases}$$

$$h(x,y) = [h(x,y) + h(-x,-y)]/2$$

$$h^{n+1} = \frac{h^{n+1}}{\int_{\Omega} h^{n+1}(x,y)dx,dy}$$

Choice of Regularization Parameters

Consider the noise-constraint minimization problem:

$$\min_{u,h} \iint |\nabla u| + \alpha |\nabla h| dxdy \quad subject\ to\ \|h * u - z\|^2 = \sigma^2$$

Using Lagrange multiplier notation,

$$f(u,h) = \iint |\nabla u| + \alpha |\nabla h| dxdy + \frac{\lambda}{2}\left(\|h * u - z\|^2 - \sigma^2\right)$$

On the other hand

$$f(u,h) = \iint \alpha_1 |\nabla u| + \alpha_2 |\nabla h| dxdy + \frac{1}{2}\|h * u - z\|^2$$

$$\Rightarrow \alpha_1 = \frac{2}{\lambda} \quad \alpha_2 = \frac{2\alpha}{\lambda}$$

Choice of Parameters

α_1 depends on the noise level

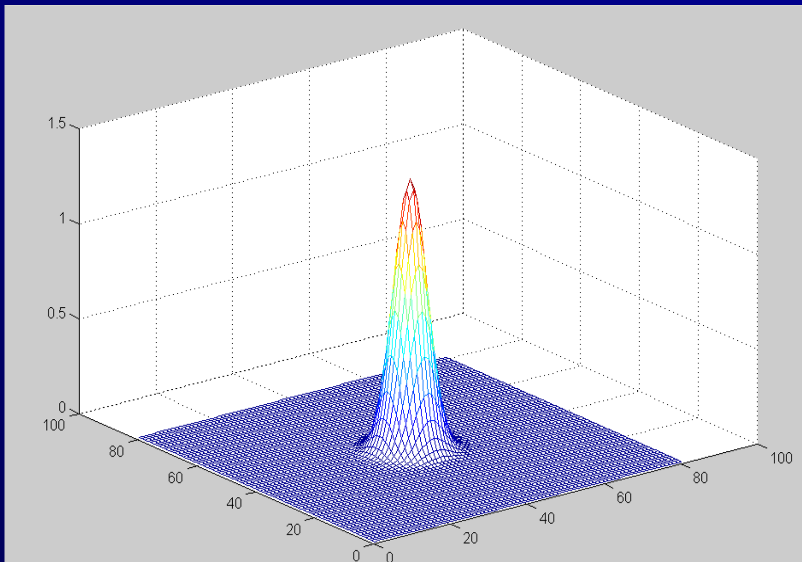
$$SNR \downarrow \Rightarrow \sigma \uparrow \Rightarrow \lambda \downarrow$$

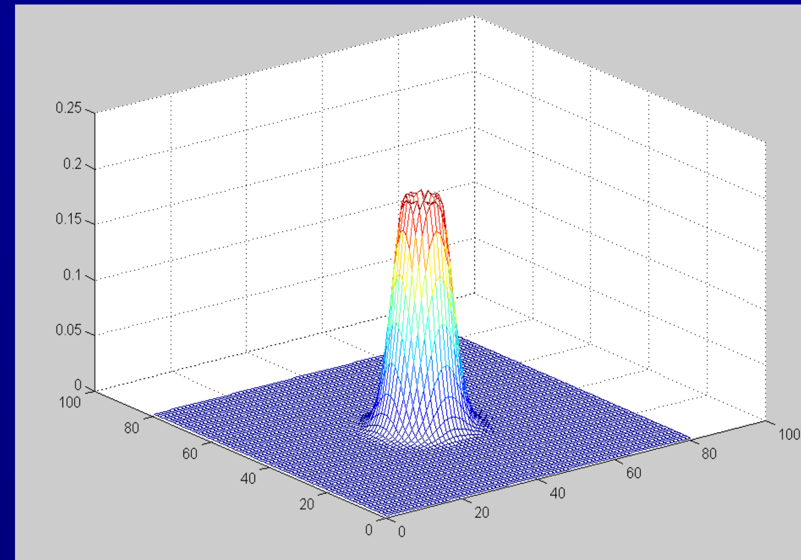
$$\alpha_1 = \frac{2}{\lambda}, \quad = C\,\sigma$$

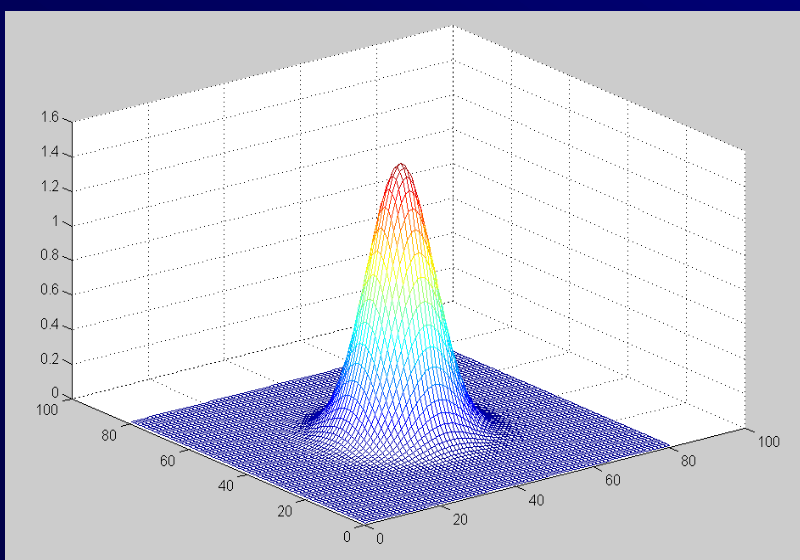
α_2 controls the support of the spread of PSF (desired deblurring)

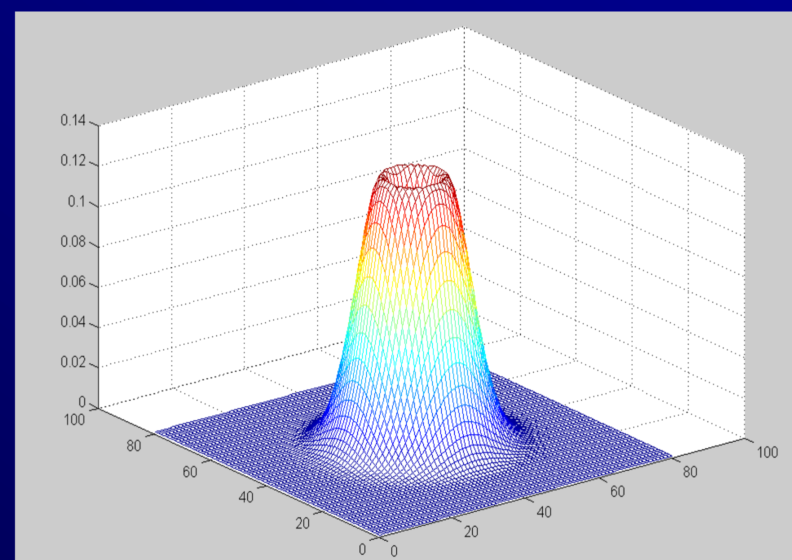
$$\alpha_2 \uparrow \Rightarrow \int |\nabla h| dxdy \downarrow, \int h dxdy = 1 \Rightarrow psf\ \ spread\ out$$

Gaussian | Total Variation









Numerical Results

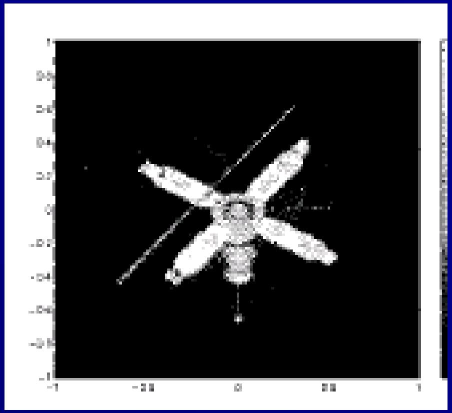
- There were 10 iterations in every FP stage.
- Within each FP iteration there is a linear system which was solved by Conjugate Gradient method (Hestenes and Stiefel 1952), which converges in a finite number of iterations.

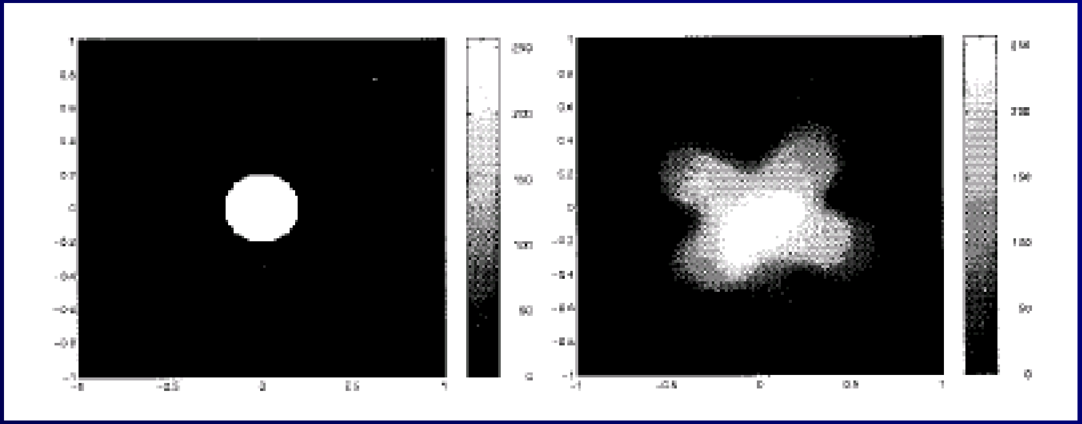
$$f(\underline{x}) = \frac{1}{2}\underline{x}^T \underline{\underline{A}}\,\underline{x} - \underline{b}^T \underline{x} - c$$

$$\min f(\underline{x}) \Leftrightarrow \underline{\underline{A}}\,\underline{x} = \underline{b} \quad \forall A \; Symmetric\ Positive\ Definite$$

Results

Test Image 127x127





Out of Focus blur blurred image

Recovered image

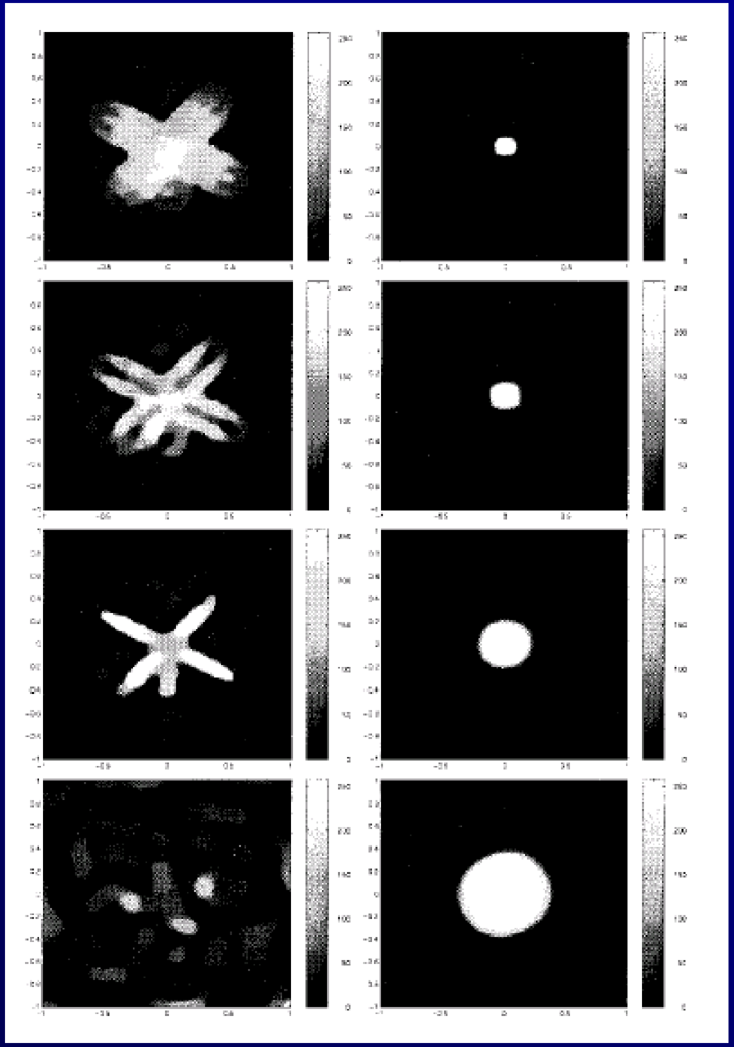
PSF

$\alpha_2 = 10^{-7}$

$\alpha_2 = 10^{-6}$

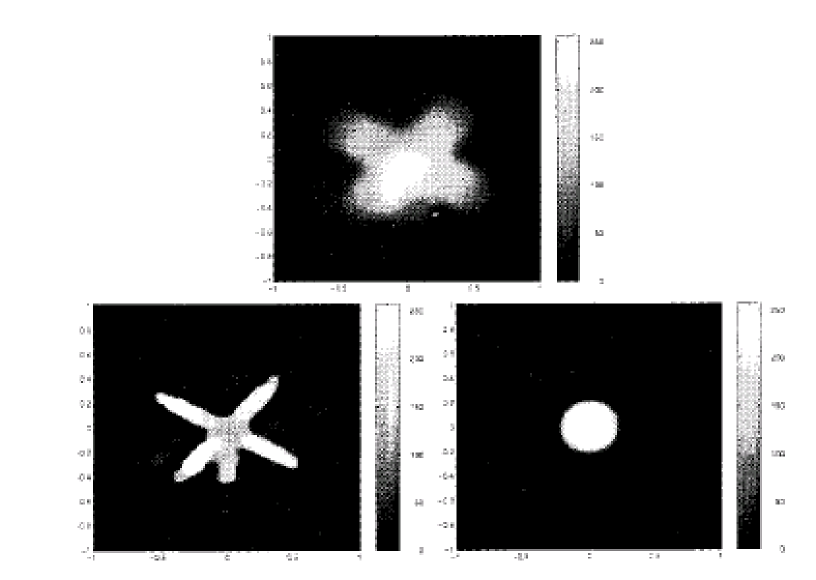
$\alpha_2 = 10^{-5}$

$\alpha_2 = 10^{-4}$

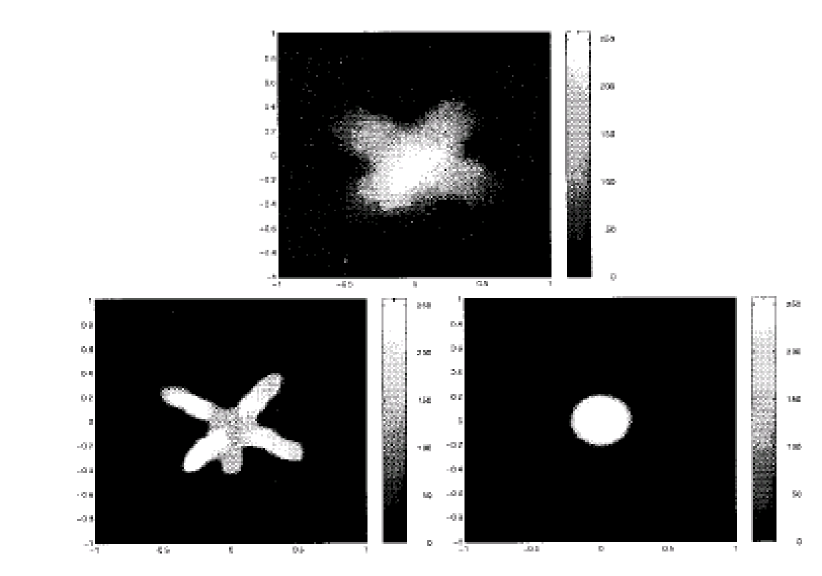


$$\alpha_1 = 2 \cdot 10^{-6}$$

Results After Three AM Iterations



$\alpha_1 = 2 \cdot 10^{-6}, \alpha_2 = 1.5 \cdot 10^{-5}, SNR = \infty$



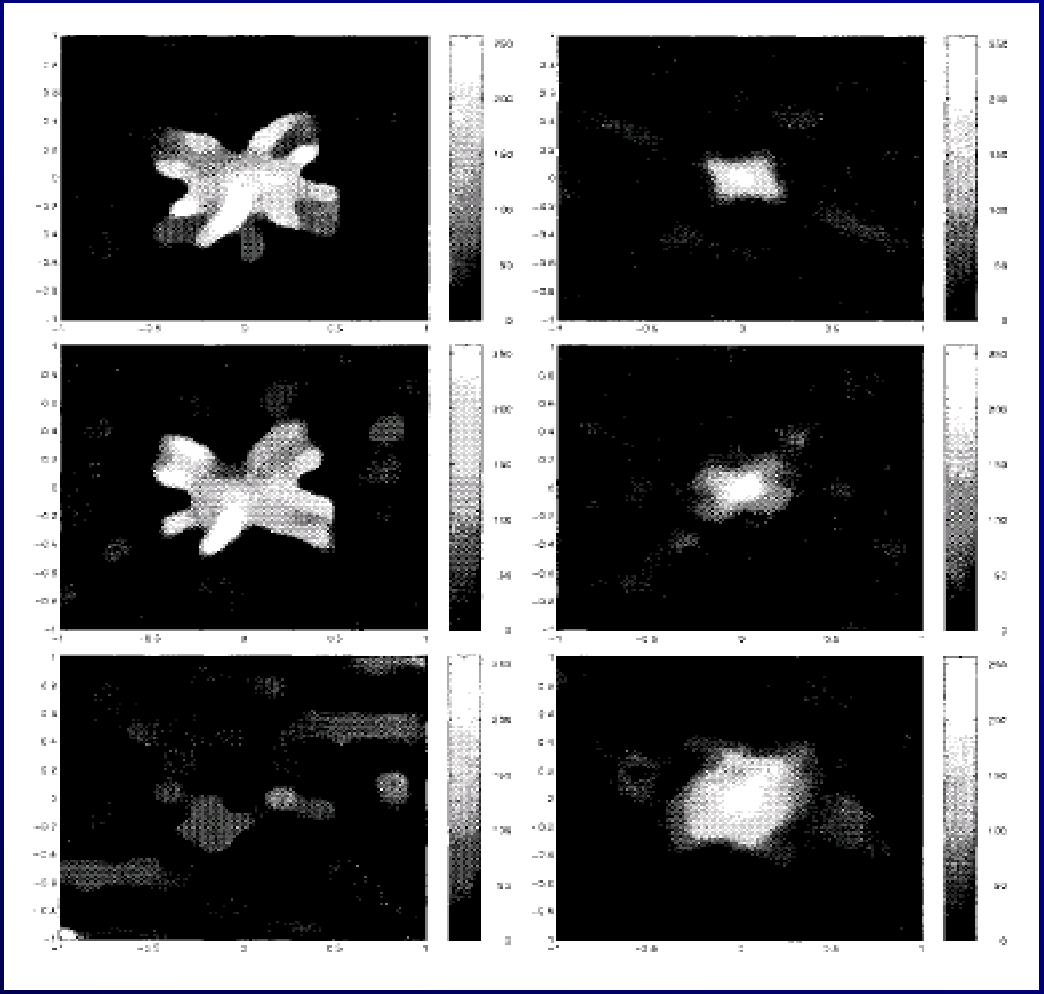
$\alpha_1 = 2 \cdot 10^{-5}, \alpha_2 = 1.5 \cdot 10^{-5}, SNR = 5$

TV Norm in u and L^2 Norm in h

$\alpha_2 = 10^{-5}$

$\alpha_2 = 10^{-4}$

$\alpha_2 = 10^{-3}$



Summary

- We used TV norm for regularizing u and h in the blind deconvolution problem
- AM/FP algorithm was proposed with choice of parameters heuristics
- Algorithm found to be robust and efficient, recovered images are as good as that recovered with the exact PSF