Euler equation

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \alpha u$$
$$\sum_{k=1}^{n} x_k \frac{\partial u}{\partial x_k} = \alpha u$$

Initial condition:

$$u(x,1) = h(x)$$
$$u(x_1, x_2, ..., x_{n-1}, +1) = h(x_1, x_2, ..., x_{n-1})$$

characteristic equations

$$\frac{\partial x_k}{\partial s} = x_k$$
$$\frac{\partial u}{\partial s} = \alpha u$$

$\frac{dx_k}{x_k} = ds$. At $s = 0$

$$x_k(s=0) = \begin{cases} t_k & k = 1, ..., n-1 \\ 1 & k = n \end{cases}$$

The solution is

$$x_k = \begin{cases} t_k e^s & k = 1, ..., n-1 \\ e^s & k = n \end{cases}$$

Then $\frac{\partial u}{\partial s} = \alpha u$ along the characteristic has the solution

$$u = e^{\alpha s} h(t_1, t_2, ... t_{n-1})$$

or

$$u(x,y) = e^{\alpha s} h(\frac{x}{y})$$
$$u(x_1, x_2, ..., x_n) = e^{\alpha s} h(\frac{x_1}{x_n}, \frac{x_2}{x_n}, ..., \frac{x_{n-1}}{x_n})$$

This has the property that

$$u(\lambda x_1, \lambda x_2, ... \lambda x_n) = \lambda u(x_1, x_2, ... x_n)$$

i.e. homogenous of degree α . Note if $\alpha < 0$ then the solution is singular at the origin.