Semi-Blind Image Deblurring in the Presence of Poisson (Photon) Noise via Mumford-Shah Regularization

Chanan Gazala

Thesis supervised by

Prof. Eli Turkel

School of Mathematical Sciences

Department of Applied Mathematics

Tel-Aviv University

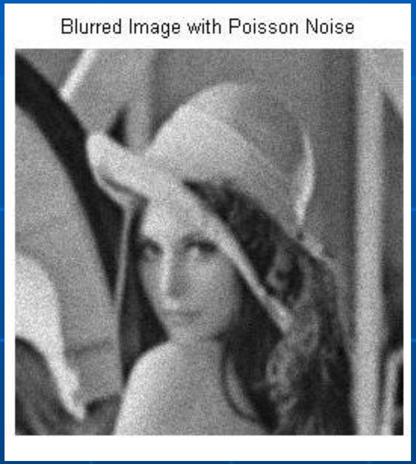
Introduction

- Our goal: to apply the variational Framework set by L. Bar, N. Sochen, N. Kiryati to handle Poisson noise as well.

- Incorporating the tasks of Poisson noise removal, semi/non blind deblurring and edge detection into a unified stochastic/ variational model.

- Problem: the data-driven nature of Poisson noise poses a major obstacle to the data-invariant Gaussian denoising models.

The Gaussian Fidelity Term is inadequate for Poisson noise



The Poisson Noise Model

For the case of additive Gaussian noise the following corruption model is considered:

$$I_{data} = I_{real} + NOISE$$

This Model is inadequate for the data-dependant nature of Poisson noise.

Recall the density function for Poisson noise,

$$P[k,\lambda] = \frac{e^{-\lambda}\lambda^{k}}{k!} \qquad k = 0,1,\ldots$$

gives the probability from 0 to 1 for the arrival of 'k' objects in a single time unit- given a mean of λ arrivals of objects per time unit.

The Poisson Noise Model

From the image processing point of view: the above objects represent light particles called *photons*.

Application: image recording devices which constructs an image based on the number of photons it was able to collect for each pixel within a limited timeslot (e.g. the time the shutter was open).

- Bright areas are expected to omit a large amount of photons.

- Darker areas have the opposite expectation.

in terms of a single pixel $I_{i,j} \Longrightarrow \lambda = I,\ k = I^0_{i,j}$

we have,

$$p\left[I^0_{i,j}, I_{i,j}\right] = p\left(I^0_{i,j} \mid I_{i,j}\right) = \frac{e^{-I_{i,j}} {I_{i,j}}^{I^0_{i,j}}}{I^0_{i,j}!}$$

The Poisson Noise Model

For the whole image *I*,

$$p\left[I^0, I\right] \stackrel{def}{=} \prod_{i,j\in\Omega} p\left(I^0_{i,j} \mid I_{i,j}\right) = \prod_{i,j\in\Omega} \frac{e^{-I_{i,j}} I_{i,j}^{I^0_{i,j}}}{I^0_{i,j}!}$$

where $\Omega \subset \mathbb{N} \times \mathbb{N}$ is the image domain.

We have the problem of maximum likelihood (ML) parameter estimation for the restoration *I*:

Instead of pursuing ***how*** the restored image can be produced, we start from the final result, specifying, via a probability function, ***what*** makes a restoration candidate *I* to be considered a good one.

-Given the restoration candidate I, we actually determine the probability that the application of the Poisson corruption process on *I* will result in the corrupted input data.

-This MLE term is also known as the *fidelity component*.

The Poisson Noise Model

Recall that in Poisson noise the mean=variance. Therefore,

$$SNR \triangleq \text{mean} \Big/ \sqrt{\text{variance}} = \lambda \Big/ \sqrt{\lambda} = \sqrt{\lambda}$$

Substituting $\lambda \triangleq I_{i,j}$

we conclude that in order to achieve a high level of SNR for our restoration I, we must collect as many of photons as possible.

- The effect of the noise will then be minimized.

Original Image

Generating a MAP Estimation Model

The fidelity term by itself is insufficient for our restoration quest.

-Problem is ill posed, i.e. the maximizing image is not unique.

- In addition, our restoration problem is an inverse one:

- Given a noisy image, determine the probability that the current candidate image is the restoration we are seeking.

- Therefore, we seek the conditional probability of $p\left(I \mid I^{0}\right)$**.**

Let us recall the Bayes equation:

$$p\left(I \mid I^{0}\right) \overset{def}{=} \frac{p\left(I^{0} \mid I\right) p\left(I\right)}{p\left(I^{0}\right)} = \frac{\prod_{i,j \in \Omega} \left\{ p\left(I^{0}_{i,j} \mid I_{i,j}\right) p\left(I_{i,j}\right) \right\}}{\prod_{i,j \in \Omega} p\left(I^{0}_{i,j}\right)}$$

Generating a MAP Estimation Model

The term, $p(I) \stackrel{def}{=} \prod_{i,j \in \Omega} p(I_{i,j})$,

is usually referred to as the *prior* or the *regularization component*.

-Defines the probability the an image I is in fact a ‘legitimate’ image.

-This can be considered as an attempt to narrow down the solution space for the estimation candidates:

- ill-posed → better-posed.

-This definition is usually elusive and application dependant, i.e. requires a-priori knowledge of the class of target image we might encounter.

-To be discussed later on…

Generating a MAP Estimation Model

After omitting the constant normalizing value of $P(I_0)$,

We are left with the maximization problem of $p(I^0 \mid I)\,p(I)$.

Rearranging the nominator of the Bays equation yields a MAP estimation problem:

$$\prod_{i,j\in\Omega}\left\{p\left(I^0_{i,j} \mid I_{i,j}\right)p\left(I_{i,j}\right)\right\} =$$

$$\left[\prod_{i,j\in\Omega}\frac{e^{-I_{i,j}}\,I_{i,j}^{\;I^0_{i,j}}}{I^0_{i,j}\,!}\right]p(I) = e^{-\int_\Omega\left(-\log\left(\frac{1}{I^0!}(I)^{I^0}\right)+I\right)dA}\cdot p(I) =$$

$$e^{-\int_\Omega\left(\log\left(I^0!\right)+I-I^0\log I\right)dA}\cdot p(I)$$

Generating a MAP Estimation Model

Omitting the constant term $\int_{\Omega} \log\left(I^0!\right) dA$ yields

$$\exp\left(-\int_{\Omega} \left(I - I^0 \log I \right) dA \right) \cdot e^{-\left(\substack{\text{some regularization factor} \\ \text{to be chosen soon...}}\right)}$$

The negative exponential form will enable use to transform the task of maximizing a probability function with the much simpler one of minimizing a functional.

The Case of a Blurred Image with Poisson Noise

We wish to incorporate together the tasks of image deblurring coupled with Poisson noise removal.

The acquired data can be seen as an image which was blurred using a Gaussian kernel and after that, underwent a Poisson (photon) noising process. mainly due to inaccuracies in the acquiring device which is based upon photon counting.

Seek the unblurred image!

The revised MAP equation now takes the form

Remove noise from the blurred image

$$p\left(I \mid I^0\right) \stackrel{def}{=} \frac{p\left(I^0 \mid I * h_\sigma\right) p\left(I\right)}{p\left(I^0\right)} =$$

$$\exp\left(-\int_\Omega \left(I * h_\sigma - I^0 \log\left(I * h_\sigma \right) \right) dA \right) \cdot e^{-\left(\text{some regularization factor to be chosen soon...}\right)}$$

Constructing the Regularization Component

When the problem is ill-posed, some of the data in the original image can never be restored.

Therefore, we must use as much a-priori knowledge as we can.

-Landscape images are expected to exhibit smoothness within their connected components. Moreover, the discontinuity hyper planes between these components should themselves be smooth.

-In astronomical imaging, on the other hand, we might encounter isolated discontinuities (e.g. remote stars) that would be considered as noise within the class of landscape images.

In this paper we will consider gradient based regularization components in which noise is described as a redundant gradient.

Total Variation (TV) Regularization

The main principle is to prefer images with the least total sum of gradient values.

Nevertheless, since this probability is multiplied by the fidelity term's probability function, the restoration candidate image still has to exhibit fidelity to the acquired noisy (and possibly blurred) input data.

This will narrow the solution space of high fidelity restorations to include only images with a minimal amount of gradients.

This approach does not discriminate between true edges and noise.

-This will be done implicitly via the fidelity term…

Tikhonov Regularization

Tichonov has offered to consider the following term

$$p_{Tikhonov}\left(I\right) = e^{-\beta \int_{\Omega} \left|\nabla I\right|^2}$$

Penalizes images whose gradients possess a high quadratic L2 norm.

Although removing most of the noise, due to the global quest for an image exhibiting small BV value, we might falsely prefer a restoration candidate with smoother edges over a better reconstruction with sharper edges.

ROF Regularization

When $\int_{\Omega} |\nabla I|^2 < \infty$ Tichonov's regularization belongs to the Sobolev space $H^1(\Omega) = W^{1,2}(\Omega)$.

-This space does not allow any discontinuities along the image's hyper surface.

-This is critical in the case of image analysis since these discontinuities are expected to appear in the boundaries between the Image's objects.

Rudin-Osher-Fetami (ROF) offered to consider a L_1 regularization component:

$$p_{TV}(I) = e^{-\beta \int_{\Omega} |\nabla I|}$$

It has been shown that when $I \in BV(\Omega)$, then there is a unique maximizer for the MAP estimation models.

- Exhibits better edge preservation.

The Mumford-Shah (MS) Regularization

ROF regularization stills fails to discriminate isolated discontinuities (noise) over smooth sets of discontinuities (edges).

A-priori knowledge:

- images in our world are piecewise smooth.
- The set of edges separating objects in an image are smooth themselves.
- The overall number of edges should be minimal
  - Edges contaminated by noise could become entangled and so longer…

How can we mathematically differentiate real edges from noise?

The Mumford-Shah (MS) Regularization

Let

K=closed edge set

Ω=open image domain

Then the Mumford-Shah (MS) regularization component is defined by

$$p(I) = p_{MS}(I,K) = e^{-\left(\beta \int_{\Omega\setminus K} |\nabla I|^2 \, dA + \alpha \int_K d\sigma\right)}$$

Piecewise smoothness

Penalty for long edges

The Mumford-Shah (MS) Regularization

Remarks:

1. We may use the better smoothing L2 norm since there are no discontinuity jumps within the connected components.
2. Since the set of edges consists of closed points,

$$\Psi\left(I,K\right)=\beta\int_{\Omega\backslash K}\left|\nabla I\right|^{2}dA+\alpha\int_{K}d\sigma=\beta\int_{\Omega}\left|\nabla I\right|^{2}dA+0$$

Therefore, to receive contribution from the edge set, we must approximate it with a continuous function, mask the image, and sum over the whole domain Ω.

From Robust Statistics to Variational Calculus

We transform our problem from a probability function maximization problem to the one of minimizing a functional:

$$\Im_{MS}\left(I,K\right)=-\log\left[p\left(I^{0}\mid I*h_{\sigma}\right)p_{MS}\left(I,K\right)\right]=$$
$$\int_{\Omega}\left(I*h_{\sigma}-I^{0}\log\left(I*h_{\sigma}\right)\right)dA+\beta\int_{\Omega\setminus K}\left|\nabla I\right|^{2}dA+\alpha\int_{K}d\sigma$$

Or, for the case of denoising only,

$$\Im_{MS}\left(I,K\right)=-\log\left[p\left(I^{0}\mid I\right)p_{MS}\left(I,K\right)\right]=$$
$$\int_{\Omega}\left(I-I^{0}\log\left(I\right)\right)dA+\beta\int_{\Omega\setminus K}\left|\nabla I\right|^{2}dA+\alpha\int_{K}d\sigma$$

A Γ-Convergent Approximation for the MS Regularization Component

The main difficulty arises from the use of Hausdorff measure of the closed set of edge points K.

Ambrosio and Tortorelli suggested an approximation for the MS prior which better suits numerical computations:

Let

$$\Psi(I,K)=\beta\int_{\Omega\backslash K}|\nabla I|^2\,dA+\alpha\int_K d\sigma$$

The unknown closed edge set is replaced by a complimentary characteristic function of *K*,

$$V(i,j)\overset{def}{=}1-\chi_K=\begin{cases}0, & (i,j)\in K\\ 1, & (i,j)\notin K\end{cases}$$

A Γ-Convergent Approximation for the MS Regularization Component

Using *V*, AT introduced a series of parameter dependant functionals

$$\Psi_\varepsilon(I,V)=\beta\int_\Omega V^2|\nabla I|^2\,dA+\alpha\int_\Omega\left(\varepsilon|\nabla V|^2+\frac{1}{4\varepsilon}(V-1)^2\right)dA$$

Although non-convex, these elliptic functionals are defined on a space of smooth functions of the same dimension which obeys the following two conditions:

1. $$\lim_{\varepsilon\to 0}\Psi_\varepsilon(I,v)=\Psi(I,K)$$

2. $$\lim_{\varepsilon\to 0}\arg\min\Psi_\varepsilon(I,v)=\arg\min\Psi(I,K)$$

in terms of Γ-convergence.

Semi-Blind Deblurring

So far, we have conveniently assumed that the blur kernel is fully known.

However, we might encounter a task of deconvolution where the kernel type is known (e.g. Gaussian, pill box) but the kernel's standard deviation is unknown→ semi-blind deblurring.

•Ambiguity problem arises:

- was the contamination a result of subsequent blur kernels or of one, smoother kernel?
- yielding an infinity amount of restoration candidates minimizing the fidelity term

$$h_{\sigma_1} * h_{\sigma_2} = h_{\sigma_1+\sigma_2}$$

Semi-Blind Deblurring

In ill-posed inverse problems we should always strive to incorporate as much a-priori knowledge as possible:

Since our application is aimed at ordinary imaging, we expect our restoration to be <u>piecewise</u> smooth.

Hence, out of all the minimizing candidates, we shall favor the smoothest.

Assume our blur kernel is a Gaussian LPF: $h_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$

The kernel smoothness component to be appended to our model is:

$$\text{RegPSF}(\sigma) = \gamma \int_\Omega \left|\nabla h_\sigma\right|^2 dA$$

Semi-Blind Deblurring

the choice of the L_2 norm of the kernel's gradient will further penalize narrow Gaussian kernels.

-Since there are no discontinuity jumps in the Gaussian kernel, its use is justified.

Our final objective functional is therefore,

$$\Im_{\varepsilon}(I,V,\sigma)=$$

$$\int_{\Omega}\left(I*h_{\sigma}-I^{0}\log\left(I*h_{\sigma}\right)\right)dA+$$

$$\beta\int_{\Omega}V^{2}\left|\nabla I\right|^{2}dA+\alpha\int_{\Omega}\left(\varepsilon\left|\nabla V\right|^{2}+\frac{1}{4\varepsilon}(V-1)^{2}\right)dA+\gamma\int_{\Omega}\left|\nabla h_{\sigma}\right|^{2}dA$$

Omit if only segmentation is desired

Omit is the blur kernel is fully known

Handcraft the functional according to the chosen inverse problem!

Minimizing the Objective Functional

After showing existence and uniqueness of a minimizer to $\mathfrak{I}_\varepsilon$

We may perform an Euler-Lagrange variation on the functional with respect to the restoration I,

$$\frac{\delta \mathfrak{I}_\varepsilon}{\delta I} = \left(1 - \frac{I_0}{(I * h_\sigma)}\right) * h_\sigma(-x, -y) - 2\beta \operatorname{Div}\left(V^2 \nabla I\right) = 0$$

or

$$\frac{\delta \mathfrak{I}_\varepsilon}{\delta I} = \frac{1}{I}(I - I_0) - 2\beta \operatorname{Div}\left(V^2 \nabla I\right) = 0$$

Data dependent!

and with respect of the estimated edge map V:

$$\frac{\delta \mathfrak{I}_{I_0}}{\delta V} = 2\beta V \left|\nabla I\right|^2 + \alpha \frac{V - 1}{2\varepsilon} - 2\varepsilon\alpha \nabla^2 V = 0$$

Minimizing the Objective Functional

Minimizing with respect of the scalar kernel parameter variable → differentiation

$$\frac{\partial \Im_{\varepsilon}}{\partial \sigma} = \int_{\Omega} \left[\left(\frac{\partial h_{\sigma}}{\partial \sigma} * I \right) \cdot \left(1 - \frac{I_0}{h_{\sigma} * I} \right) + \gamma \frac{\partial}{\partial \sigma} \left| \nabla h_{\sigma} \right|^2 \right] dA = 0$$

Convex and bounded from below with respect of I or V.

Not convex with respect of σ:

- convergence to a local minimum is therefore possible.

General solution framework: Alternate minimization.

Numerical Solution

General Solution Algorithm

Initialize:

- MaxIterationsNum
- Tolerance_for_convergence
- α, β, γ
- start_sigma $\ll 1$
- sigma=start_sigma
- sigma_convergence_flag=0
- *I=I0*
- V=ones(M,N) → assume fully smooth at initialization!

General Solution Algorithm- continued

Loop for MaxIterationsNum:

1. solve linear system for V.

2. solve nonlinear system for I.

3. If sigma_convergence_flag=0:

    a. old_sigma=sigma

    b. sigma = find zero crossing of $\frac{\partial F_{\varepsilon}}{\partial \sigma}$

    c. if $\left| \text{sigma} - \text{old\_sigma} \right|_2 < \text{Tolerance\_for\_convergence}$
    then sigma_convergence_flag=1

4. If $\left| I^0 - A[I] \right|_2 < \text{Tolerance\_for\_convergence}$ then exit

Discrete Equation for the Kernel's Parameter

$$F(\sigma) = \frac{\partial \Im_\varepsilon}{\partial \sigma} =$$

$$\left(I^n * \sum_{x=-M}^{M} \sum_{y=-M}^{M} \left[\frac{1}{2\pi\sigma^2} \left(\frac{x^2+y^2}{\sigma^3} - \frac{2}{\sigma} \right) e^{\frac{-\left(x^2+y^2\right)}{2\sigma^2}} \right] \right) \cdot \left(1 - \frac{I_0}{h_\sigma * I^n} \right) +$$

$$2\gamma \sum_{x=-M}^{M} \sum_{y=-M}^{M} \left[\frac{x^2+y^2}{\pi^2} \left(\frac{x^2+y^2}{\sigma^{11}} - \frac{4}{\sigma^9} \right) e^{\frac{-\left(x^2+y^2\right)}{\sigma^2}} \right] = 0$$

The Iterative Approach

Solve using highest resolution only

nonlinear PDE for the restoration is solved via gradient descent:

$$I^{n+1} = I^{n} + \Delta t \left[-\left(1 - \frac{I^{0}}{I^{n} + \varepsilon} \right) + \left(\Delta_{+}^{x}\left(v^{2} \Delta_{-}^{x} \right) + \Delta_{+}^{y}\left(v^{2} \Delta_{-}^{y} \right) \right) I^{n} \right]$$

or

$$I^{n+1} = I^{n} + \Delta t \left[-\left(1 - \frac{I^{0}}{\left(I^{n} * h_{\sigma} \right) + \varepsilon} \right) * h_{\sigma}(-x, -y) + \left(\Delta_{+}^{x}\left(v^{2} \Delta_{-}^{x} \right) + \Delta_{+}^{y}\left(v^{2} \Delta_{-}^{y} \right) \right) I^{n} \right]$$

- Slow though robust.
- Requires careful choice of the Δt parameter

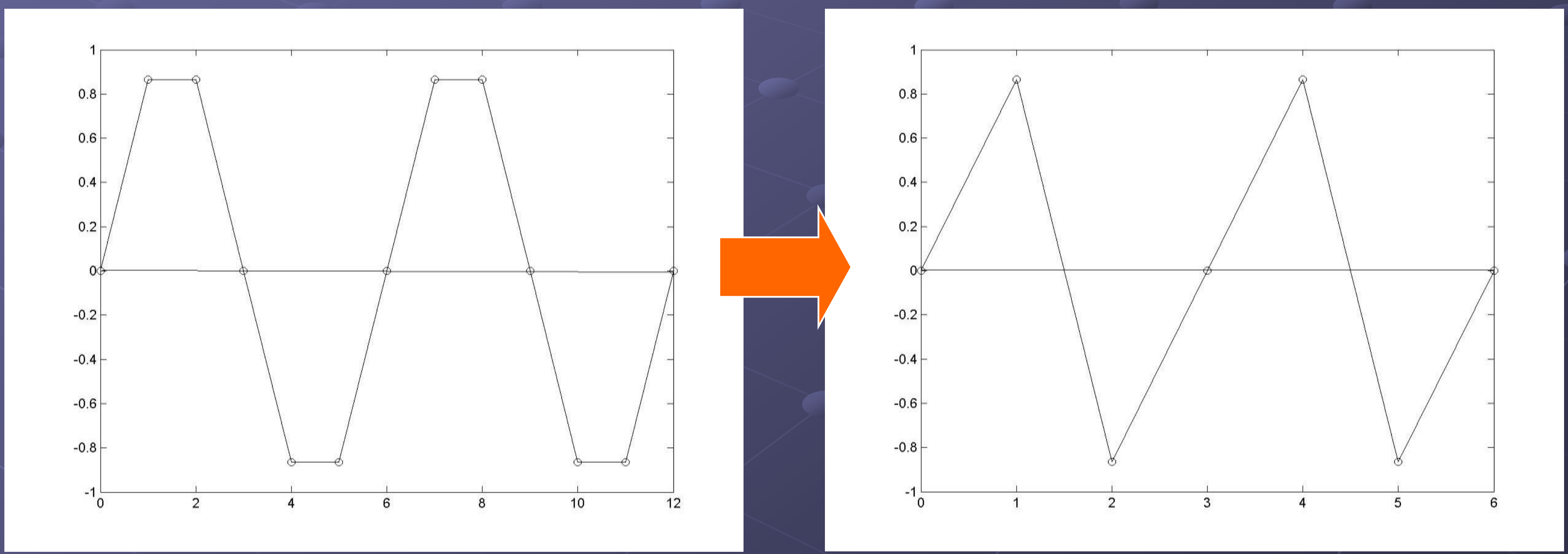
Linear PDE for the edge map V is solved via GMRES.

The Multigrid Framework

most iterative approaches posses the following property:

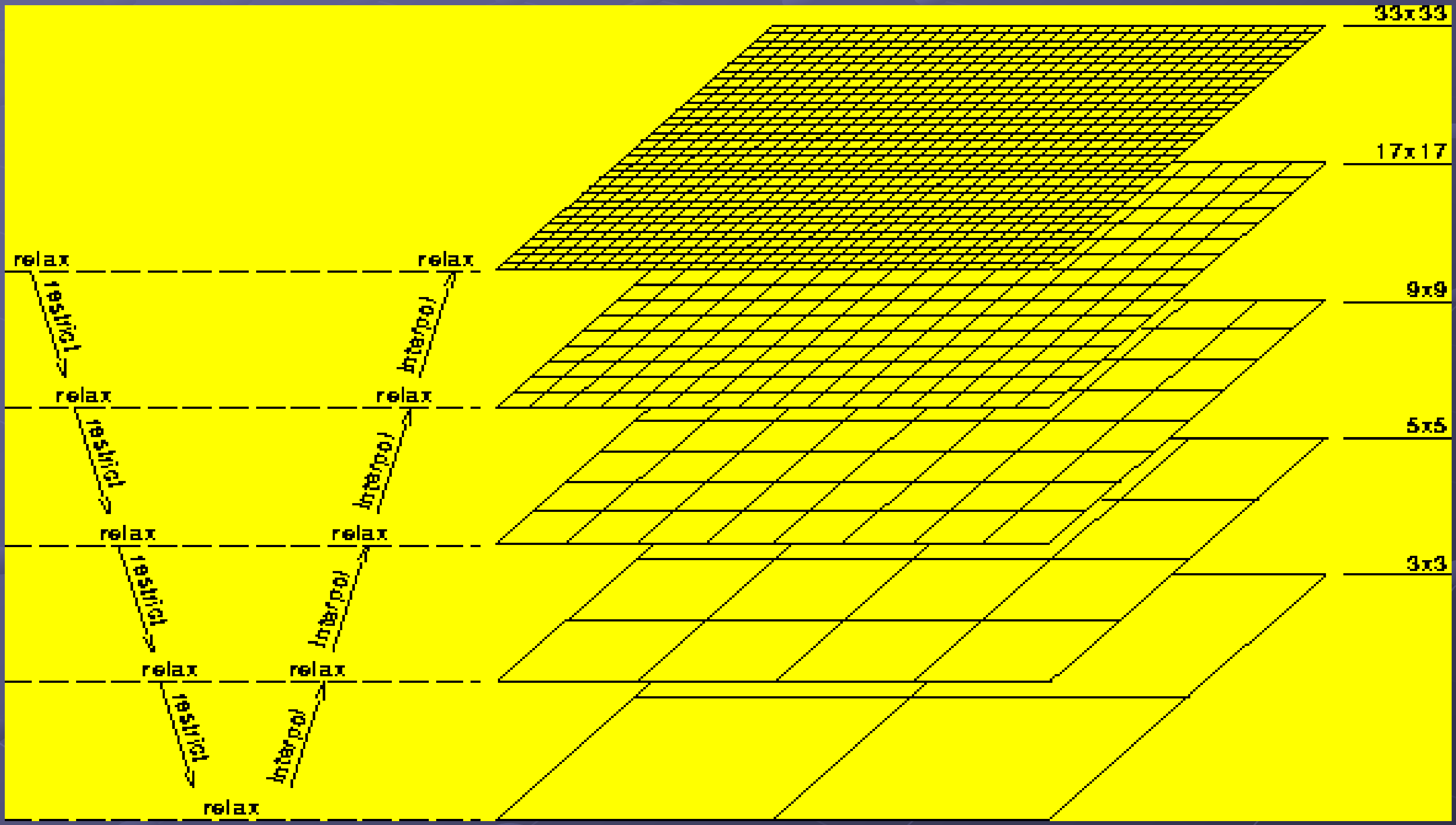
As they relax over the estimation, the high frequencies of the error will be smoothed, while the low frequencies remain virtually unchanged.

Still, this property could be a ‘blessing in disguise’…



The V-Cycle Scheme

Fine grid's smooth error components → coarse grid's oscillatory error components!



The V-Cycle Scheme

Requires the following fundamental operations:

- **Restriction:**

$$v_j^{2h} = \frac{1}{4}\left(v_{2j-1}^h + 2v_{2j}^h + v_{2j+1}^h\right)$$

- **Prolongation:**

$$v_{2j}^h = v_j^{2h}$$

$$v_{2j+1}^h = \frac{1}{2}\left(v_j^{2h} + v_{j+1}^{2h}\right)$$

$$0 \le j \le \frac{n}{2} - 1$$

- **Relaxation/ smoothing scheme**
- **Residual formula**

Half weighting

Bilinear interpolation

The V-Cycle Scheme-
Linear Edge-Map PDE

Algorithm *V-cycle^h (v^h, f^h):*

Initialize:

• *pre*– number of smoothing iterations performed on a grid before restricting the signal onto a coarser one. To be referred to as *presmoothing*.

• *post*– number of smoothing iterations performed on a grid after a correction has been interpolated from the coarser grid and added to the current signal. To be referred to as *postsmoothing*.

- $V^h \equiv V^0$ initial guess.
- $f^h \equiv \left[\frac{\alpha}{2\varepsilon}\right]_{i,j}^{h}$ initial right hand side.

The V-Cycle Scheme-
Linear Edge-Map PDE

Algorithm (continued):

1. perform a *pre* number of relaxation steps thus obtaining a solution to

   $$A^h u^h = f^h$$

   .
2. If we have reached the coarsest grid, jump to step 4. Otherwise, perform the following assignments:

   $$f^{2h} \leftarrow R(f^h - A^h v^h)$$

   $$v^{2h} = 0$$

   $$v^{2h} \leftarrow Vcycle^{2h}(v^{2h}, f^{2h})$$

3. After retrieving the approximation for the error v^{2h} perform a correction by $v^h \leftarrow v^h + P v^{2h}$

.4. If at the coarsest grid, solve 'exactly'. Otherwise, relax post times on $A^h u^h = f^h$ and return recursively.

The V-Cycle Scheme-
Linear Edge-Map PDE

Discrete Equations:

Smoother: symmetric Gauss-Seidel (SGS)

$$v_{i,j}^{*}=\frac{\left[2\alpha\varepsilon\left(v_{i+1,j}^{old}+v_{i-1,j}^{*}+v_{i,j+1}^{old}+v_{i,j-1}^{*}\right)+\frac{\alpha}{2\varepsilon}\right]}{2\beta\left[\left(\frac{I_{i+1,j}-I_{i-1,j}}{2}\right)^{2}+\left(\frac{I_{i,j+1}-I_{i,j-1}}{2}\right)^{2}\right]+\frac{\alpha}{2\varepsilon}+8\alpha\varepsilon}$$

$i=1,2,\ldots,M$

$j=1,2,\ldots,N$

$$v_{i,j}^{new}=\frac{\left[2\alpha\varepsilon\left(v_{i+1,j}^{*}+v_{i-1,j}^{new}+v_{i,j+1}^{*}+v_{i,j-1}^{new}\right)+\frac{\alpha}{2\varepsilon}\right]}{2\beta\left[\left(\frac{I_{i-1,j}-I_{i+1,j}}{2}\right)^{2}+\left(\frac{I_{i,j-1}-I_{i,j+1}}{2}\right)^{2}\right]+\frac{\alpha}{2\varepsilon}+8\alpha\varepsilon}$$

$i=M,M-1,\ldots,1$

$j=N,N-1,\ldots,1$

The V-Cycle Scheme-
Linear Edge-Map PDE

Discrete Equations:

Operator *L=Av*:

$$\left(Av^n\right)_{ij} = v_{i,j}^n \left(2\beta\left[\left(\frac{I_{i+1,j} - I_{i-1,j}}{2}\right)^2 + \left(\frac{I_{i,j+1} - I_{i,j-1}}{2}\right)^2\right] + \frac{\alpha}{2\varepsilon} \right) - 2\alpha\varepsilon\left(v_{i+1,j}^n + v_{i-1,j}^n + v_{i,j+1}^n + v_{i,j-1}^n - 4v_{i,j}^n\right)$$

The V-Cycle Scheme-
Nonlinear Restoration PDE

When confronted by a nonlinear PDE the linear V-Cycle scheme cannot be used:

$$r = A(u) - A(v) \neq A(u - v) = A(e)$$

Strategy: instead of approximating the error, we solve for the *full approximation of the exact(!!!) solution …*

Revised residual equation, $$A^{2h}(v^{2h} + e^{2h}) - A^{2h}(v^{2h}) = r^{2h}$$

Using the identity $$r^{2h} = Rr^h = R(f^h - A^h(v^h))$$

We obtain the final residual relaxation formula

$$\boxed{A^{2h}(\underbrace{Rv^h + e^{2h}}_{u^{2h}}) = A^{2h}(Rv^h) + \underbrace{R(f^h - A^h(v^h))}_{r^{2h}}}$$

The V-Cycle Scheme-
Nonlinear Restoration PDE

After the iterative solver converges, e^{2h} is obtained by

$$e^{2h} = u^{2h} - Rv^h$$

and interpolated back to the finer grid by

$$v^h \leftarrow v^h + Pe^{2h} = v^h + P(u^{2h} - Rv^h)$$

Known as the ***full approximation scheme* (FAS),** it can be incorporated into any of the various multigrid frame works

- e.g. V-Cycle, full multigrid.

Superior over other linearization schemes (e.g. Newton's MG) since it solves the original nonlinear discrete problem…

The V-Cycle Scheme-
Nonlinear Restoration PDE

Discrete Equations:

Smoother: damped/weighted Jacobi

$$I^{*} = I^{old} - \frac{\overbrace{-\frac{I^0}{\left(\left(I^{old} * h_\sigma\right) + \varepsilon\right)} * h_\sigma(-x,-y) - 2\beta \mathrm{Div}\left(V^2 \nabla I^{old}\right)}^{A\left(I^{old}\right)} - \overbrace{\left(-1 * h_\sigma(-x,-y)\right)}^{\text{right hand side}}}{\underbrace{\left[\left(\frac{I^0}{\left(\left(I^{old} * h_\sigma\right)^2 + \varepsilon\right)} * h_\sigma(-x,-y)\right) * h_\sigma(-x,-y) + \varepsilon\right]}_{A'\left(I^{old}\right)}}$$

$$I^{(n+1)} = (1-\omega) I^{(n)} + \omega I^{(*)}$$

$$\omega = 0.8,\ \varepsilon \ll 1,\ \mathrm{Div}\left(V^2 \nabla I^{old}\right) = \Delta_+^x\left(v^2 \Delta_-^x\right) + \Delta_+^y\left(v^2 \Delta_-^y\right)$$

Computational Results

Poisson Denoising of an Unblurred Image

Original Image

n Image with Poisson Noise

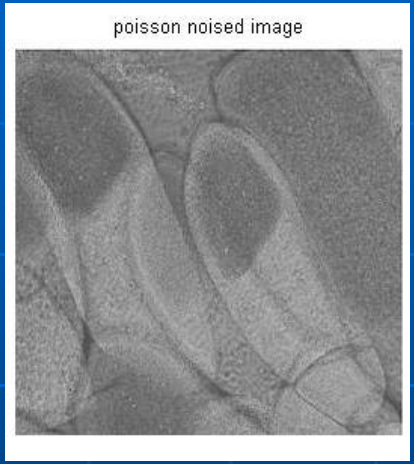
Image with Gaussian Noise

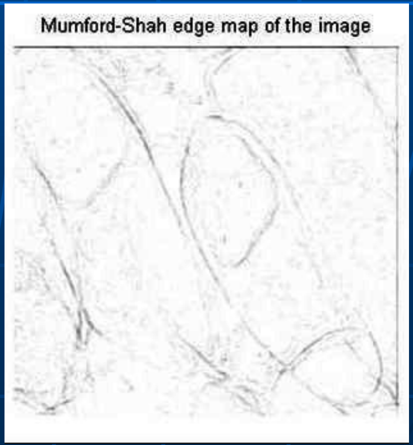
ROF Restored Image

Mumford-Shah Restored Image

Mumford-Shah Edge Map

Poisson Denoising of an Unblurred Image





Non-Blind Deblurring In the Presence of Poisson Noise

Blurred Image-sigma=1.5

Blurred Image with Poisson Noise

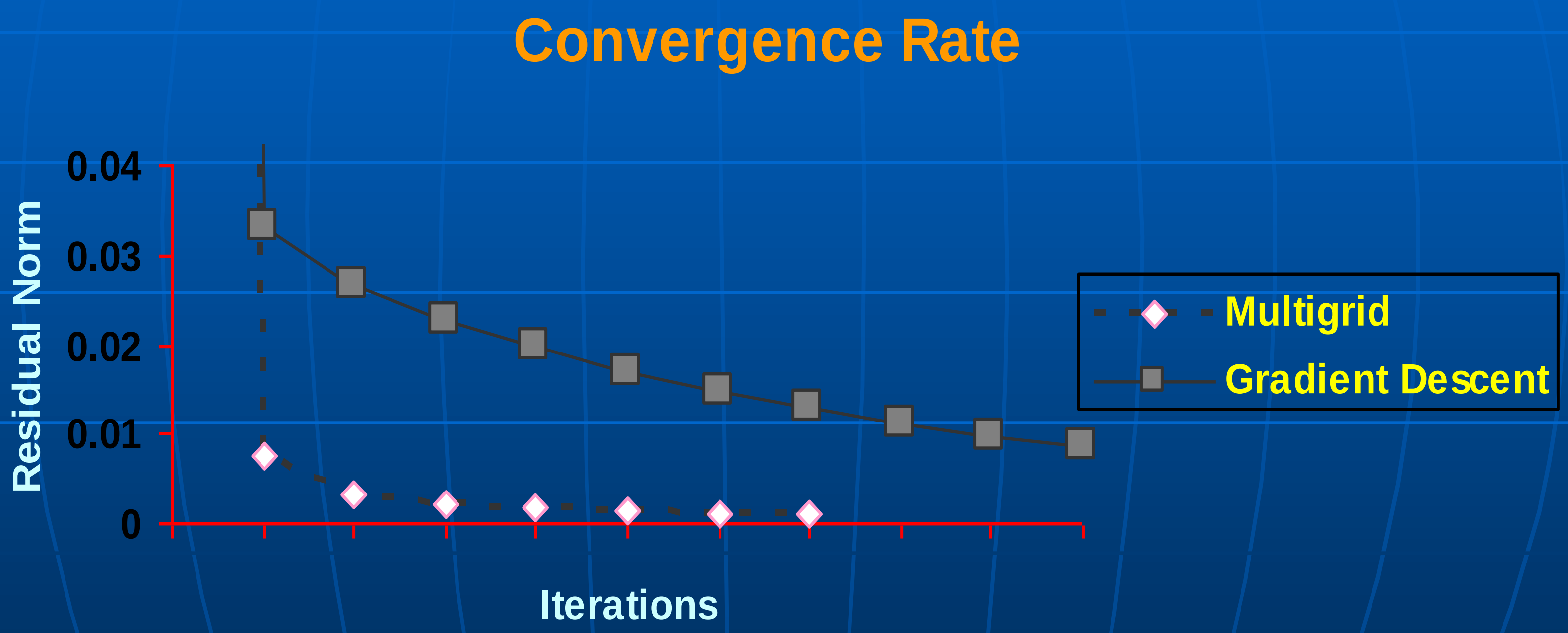
MS-GD Poisson Restored Image

MS-GD Edge Map

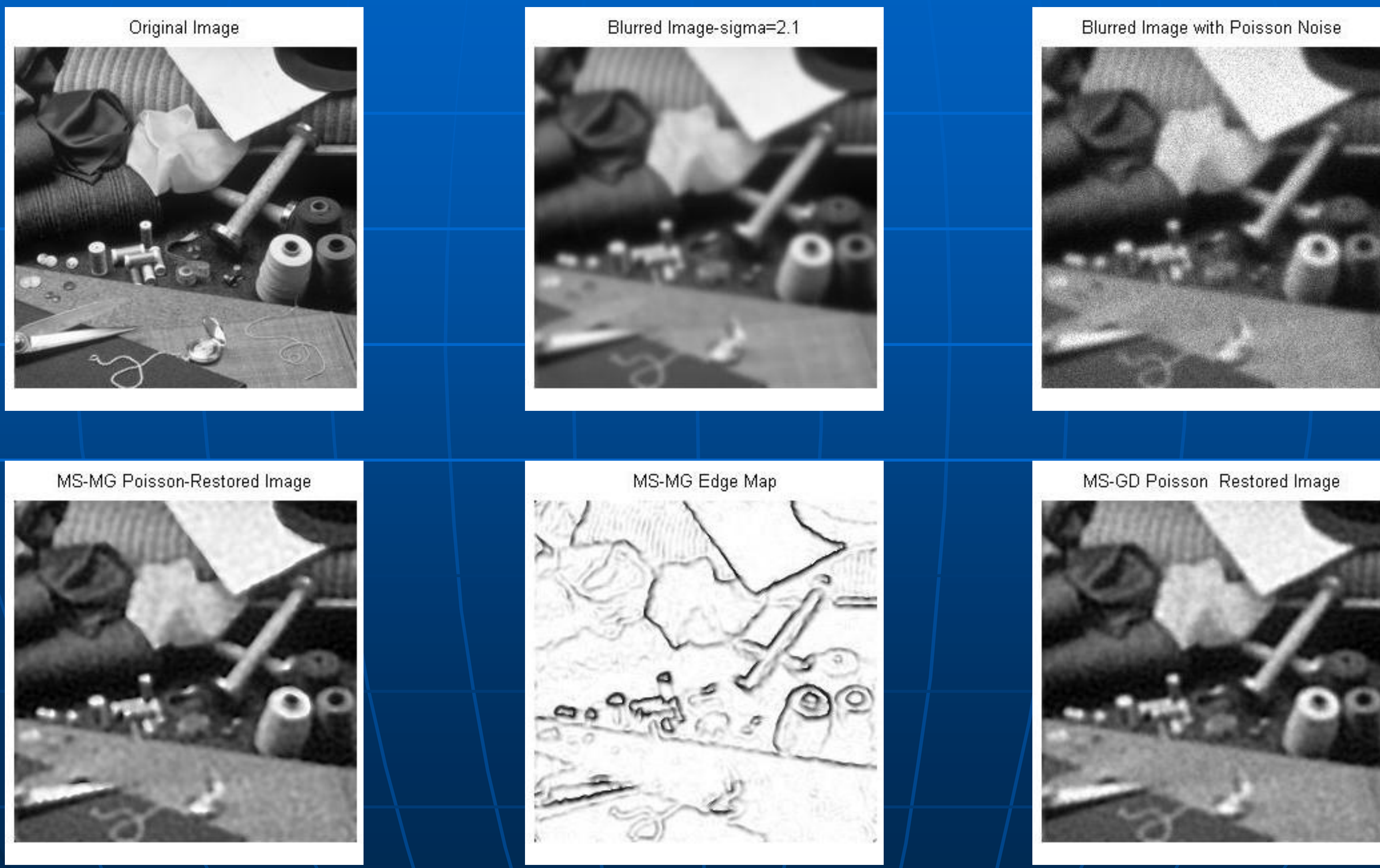
MS-MG Poisson-Restored Image

MS-MG Edge Map

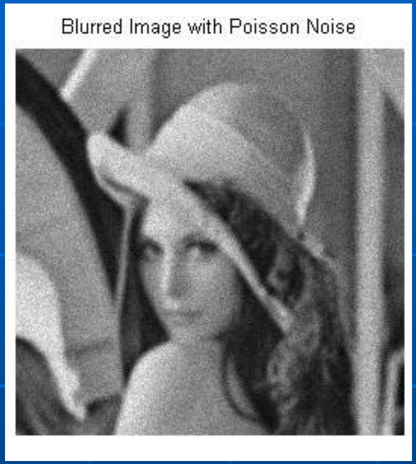
Non-Blind Deblurring In the Presence of Poisson Noise

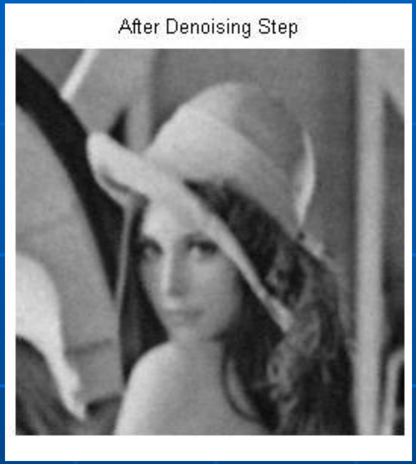


Non-Blind Deblurring In the Presence of Poisson Noise



Special Case: The Denoising and Deblurring as a Two Step Problem





Outcome:

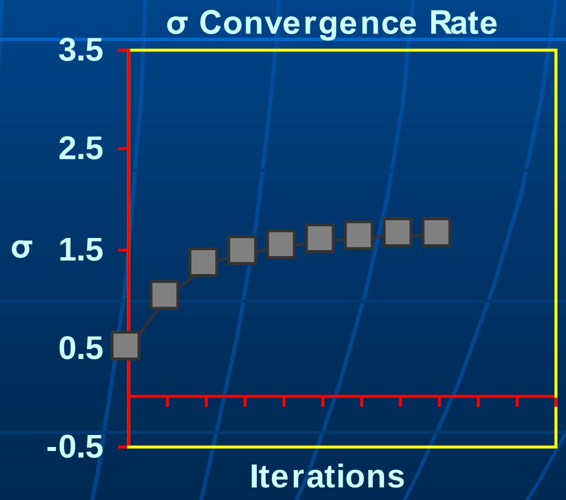
we may assume that both the restoration systems share the fixed point property with respect to the tasks of denoising and deblurring.

Semi-Blind Deblurring In the Presence of Poisson Noise

Actual parameter: 1.6

Initial parameter: 0.5

Estimated parameter: 1.65



Semi-Blind Deblurring In the Presence of Poisson Noise

Actual parameter: 2

Initial parameter: 0.5

Estimated parameter: 1.913