Question

What is Mathematical Morphology ?

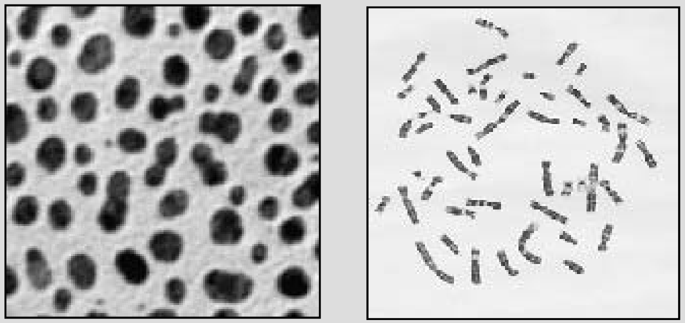
An (imprecise) Mathematical Answer

A mathematical tool for investigating geometric structure in binary and grayscale images.

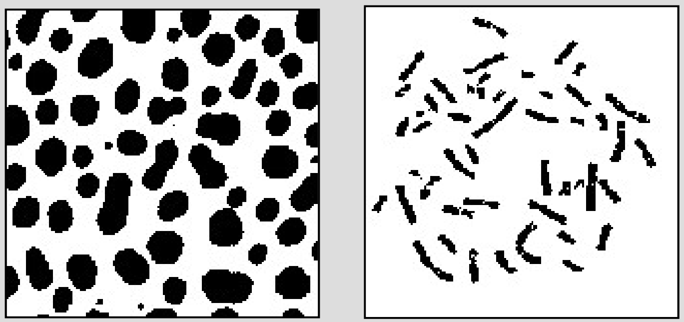
Shape Processing and Analysis

- Visual perception requires transformation of images so as to make explicit particular shape information.
- Goal: Distinguish meaningful shape information from irrelevant one.
- The vast majority of shape processing and analysis techniques are based on designing a shape operator which satisfies desirable properties

Example



Grayscale Images

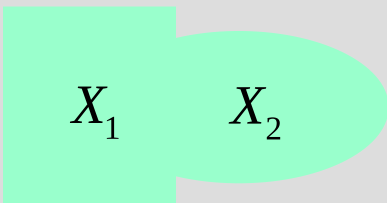


Binary Images

- Image analysis consists of obtaining measurements characteristic to images under consideration.
- Geometric measurements (e.g., object location, orientation, area, length of perimeter)

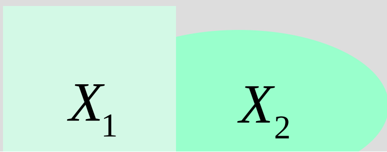
Morphological Shape Operators

- Objects are **opaque** and shape information **is not** additive !!
- Shapes are usually combined by means of:
  - **Set Union** (overlapping objects):



$$X_1 \bigcup X_2$$

  - **Set Intersection** (occluded objects):



$$X_2 \setminus X_1 = X_1^c \bigcap X_2$$

Morphological Shape Operators

- Shape operators should distribute over set-unions and set-intersections (a type of "linearity") !

Morphological Dilation	$\Psi_\delta(X_1 \cup X_2) = \Psi_\delta(X_1) \cup \Psi_\delta(X_2)$
Morphological Erosion	$\Psi_\varepsilon(X_1 \cap X_2) = \Psi_\varepsilon(X_1) \cap \Psi_\varepsilon(X_2)$

Morphological Operators

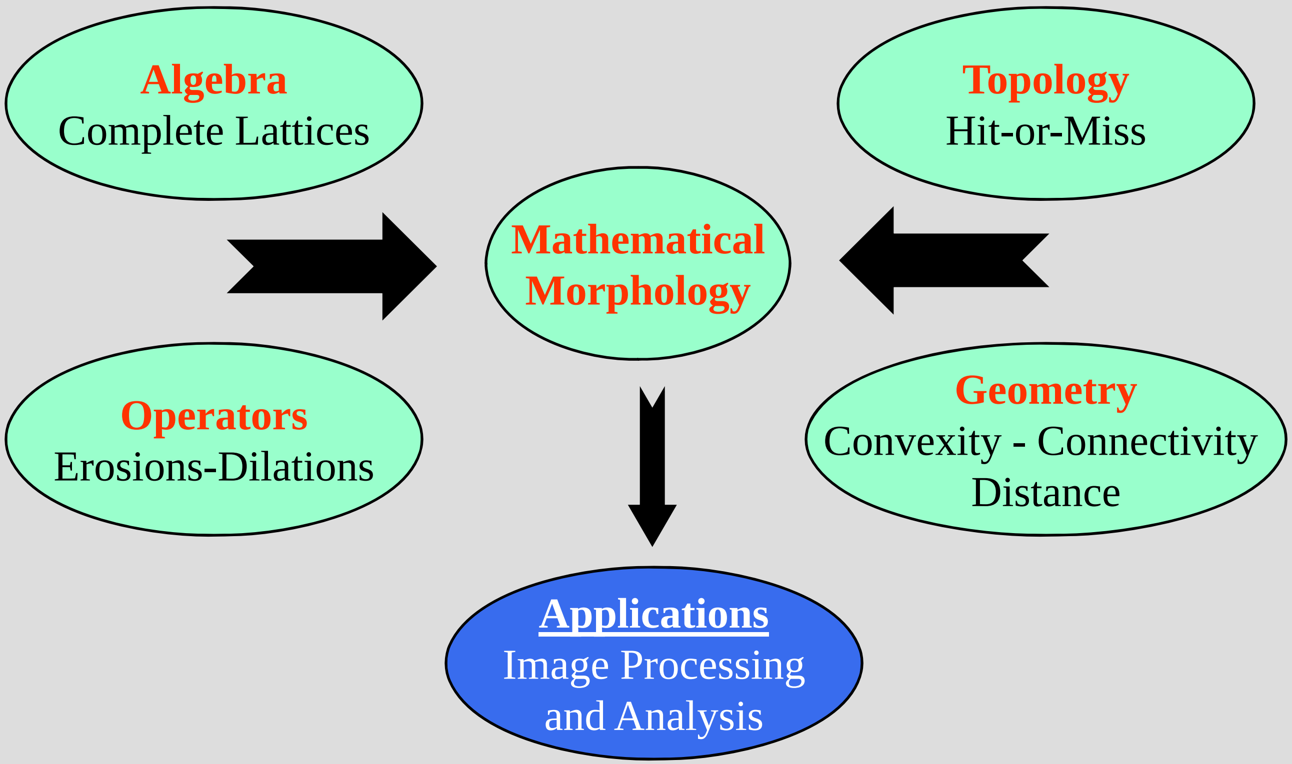
- Erosions and dilations are the most elementary operators of mathematical morphology.

- More complicated morphological operators can be designed by means of combining erosions and dilations.

Question

What is Mathematical Morphology ?

A (precise) Mathematical Answer

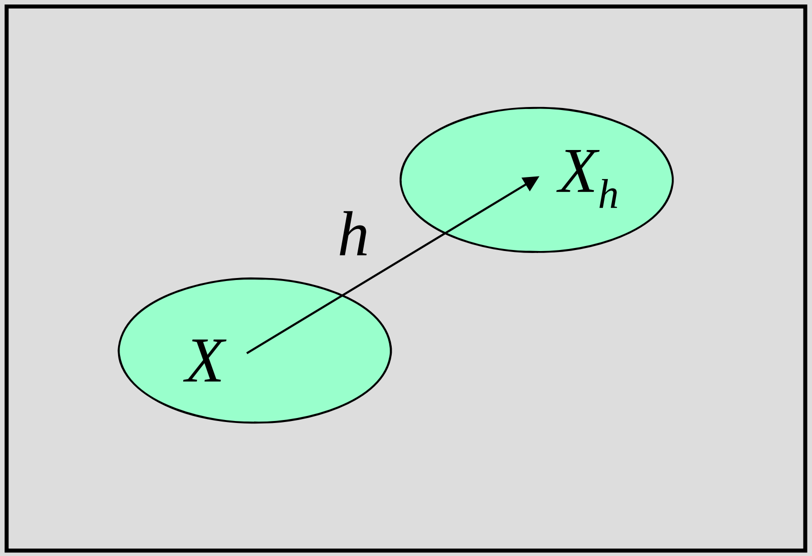


A mathematical tool that studies operators

Some History

- George Matheron (1975) *Random Sets and Integral Geometry*, John Wiley.
- Jean Serra (1982) *Image Analysis and Mathematical Morphology*, Academic Press.
- Petros Maragos (1985) *A Unified Theory of Translations-Invariant Systems with Applications to Morphological Analysis and Coding of Images*, Doctoral Thesis, Georgia Tech.

Translation Invariant Operators



$$\Psi(X_h) = [\Psi(X)]_h$$

Morphological Erosion

“LINEARITY”

$$\Psi_\varepsilon(X_1 \cap X_2) = \Psi_\varepsilon(X_1) \cap \Psi_\varepsilon(X_2)$$

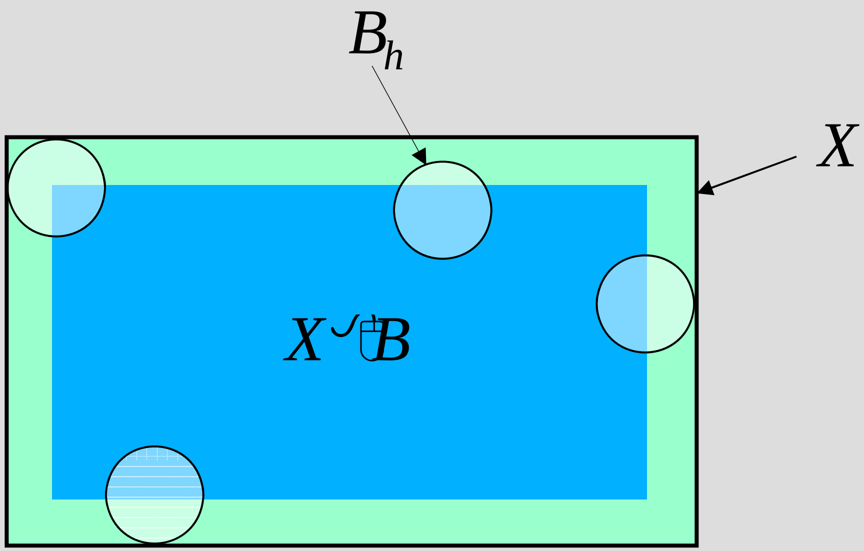
TRANSLATION INVARIANCE

$$\Psi_\varepsilon(X_h) = [\Psi_\varepsilon(X)]_h$$

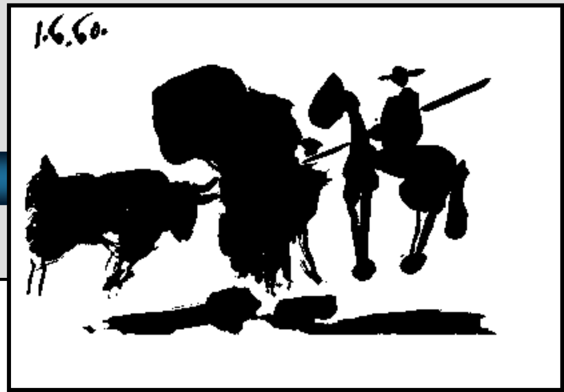
$$\Psi_\varepsilon(X) = X \ominus \check{B} = \{h \mid B_h \subseteq X\}$$

Morphological Erosion

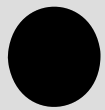
$$\Psi_{\varepsilon}(X) = X \ominus \check{B} = \{ h \mid B_h \subseteq X \}$$



Morphological Erosion



Structuring Element



Morphological Dilation

"LINEARITY"

$$\Psi_\delta(X_1 \cup X_2) = \Psi_\delta(X_1) \cup \Psi_\delta(X_2)$$

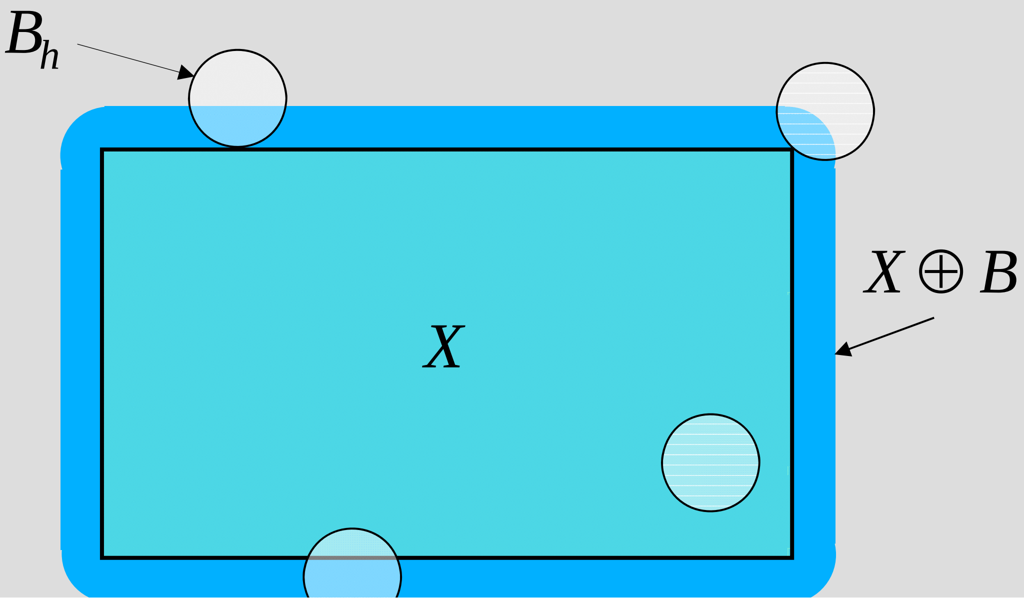
TRANSLATION INVARIANCE

$$\Psi_\delta(X_h) = [\Psi_\delta(X)]_h$$

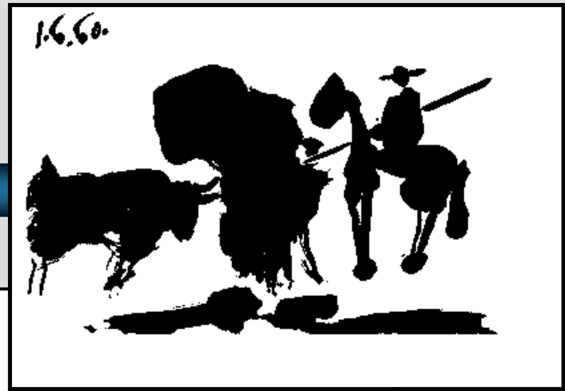
$$\Psi_\delta(X) = X \oplus B = \{h \mid B_h \cap X \neq \emptyset\}$$

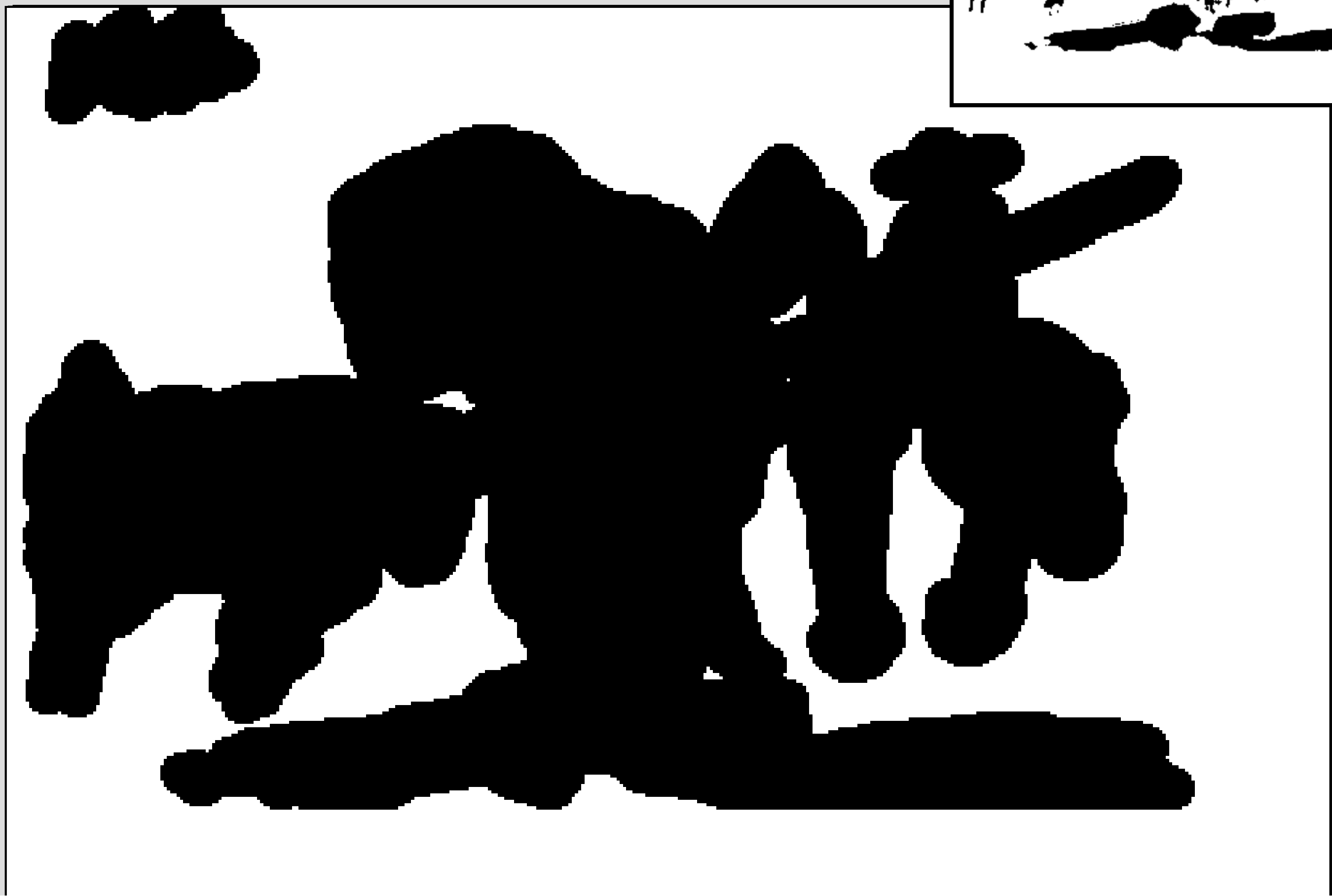
Morphological Dilation

$$\Psi_\delta(X) = X \oplus B = \{h \mid B_h \cap X \neq \emptyset\}$$

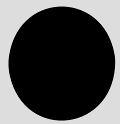


Morphological Dilation





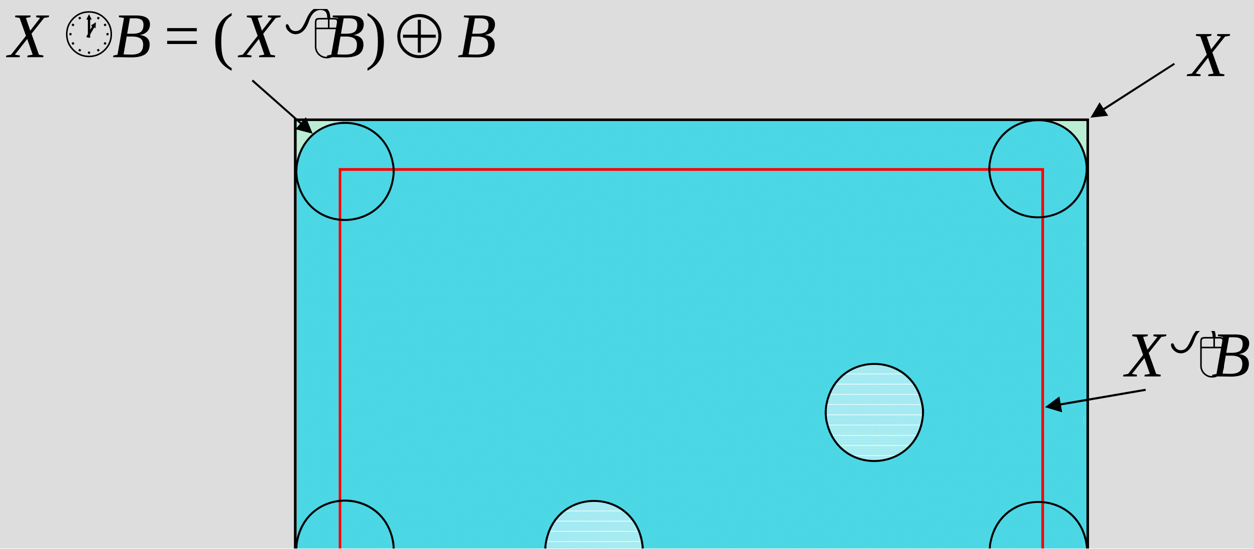
Structuring Element



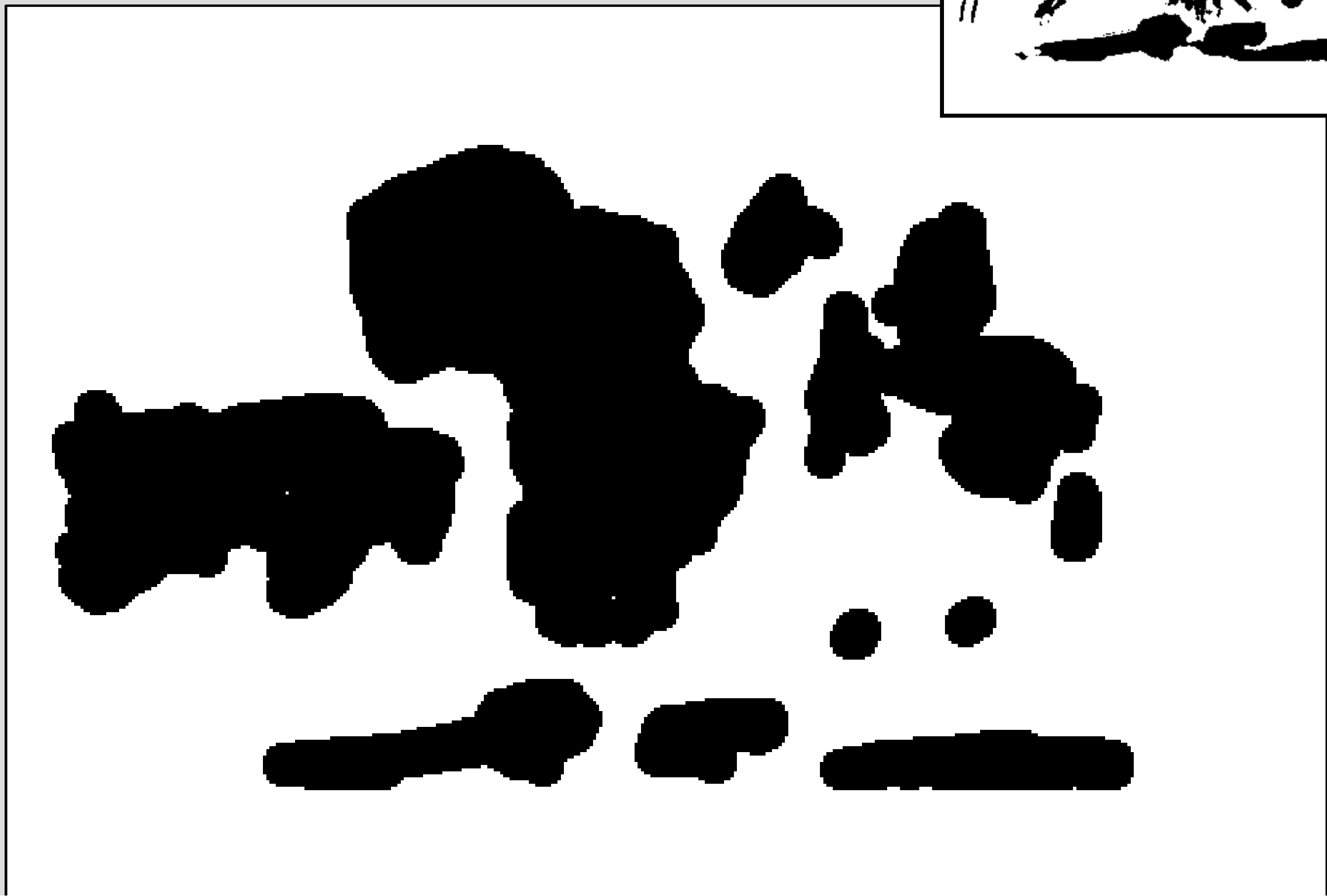
Morphological Opening

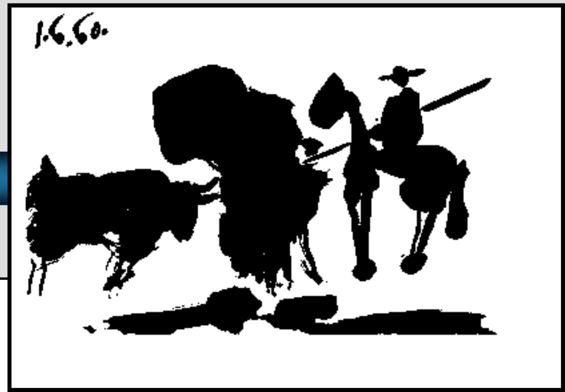
$$X \circ B = (X \ominus B) \oplus B$$

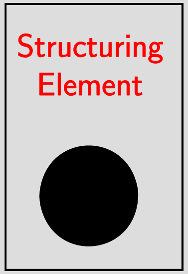
$$= \bigcup \{ B_h \mid B_h \subseteq X \}$$



Morphological Opening





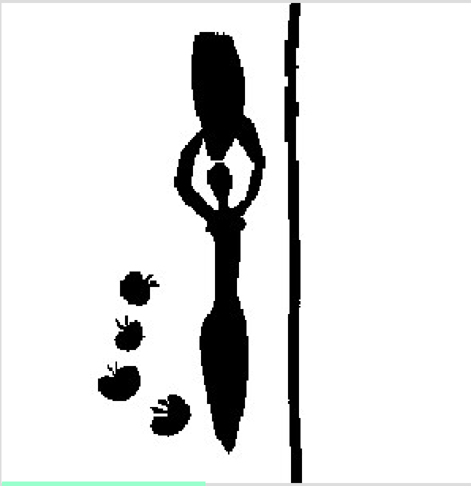


Morphological Opening

- Is a smoothing filter !
- Amount and type of smoothing is determined by the shape and size of the structuring element.
- Approximates a shape from below, since

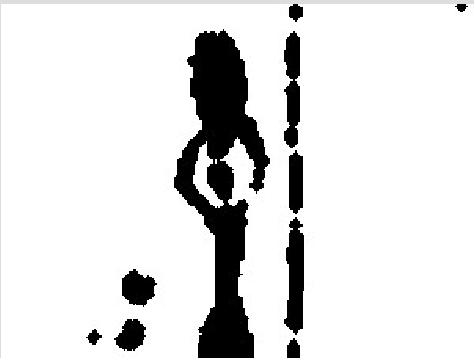
$$X \circ B \subseteq X$$

Filtering Example



ORIGINAL

DEGRADED



Henri Matisse, *Woman with Amphora and Pomegranates*, 1952

An Important Result

Increasing Operator

$$X_1 \subseteq X_2 \;\Rightarrow\; \Psi(X_1) \subseteq \Psi(X_2)$$

+

Translation Invariant Operator

$$\Psi(X_h) = [\Psi(X)]_h$$

⬇

$$\Psi(X) = \bigcup_B X \ominus B = \bigcap_B X \oplus B$$!!

Main Idea

- Examine the geometrical structure of an image by matching it with small patterns at various locations.
- By varying the size and shape of the matching patterns, called structuring elements, one can extract useful information about the shape of the different parts of the image and their interrelations.
- Results in image operators which are well suited for the analysis of the geometrical and topological