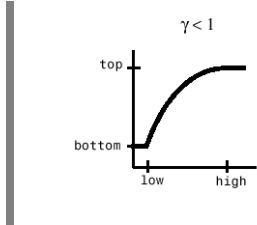


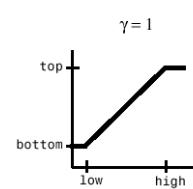


Non-linear LUT (1): Gamma Correction

brighten



darker



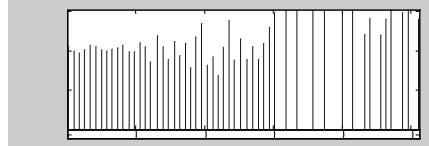
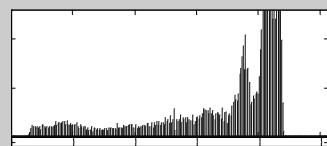
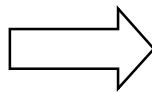
$$I_{out} = 255 \cdot \left(\frac{I - I_{min}}{I_{max} - I_{min}} \right)^{\gamma}$$

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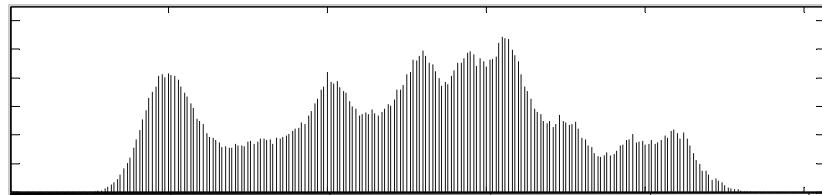
Histogram equalization (2)

Adam





Histograms



Who invented it and why?

First introduced by **Abu Yusuf Yaqub ibn Ishaq al-Sabbah Al-Kindi** as a tool for deciphering the substitution code - a breakthrough at that time, 850 AD. (see: simon singh "the code book" 1999)

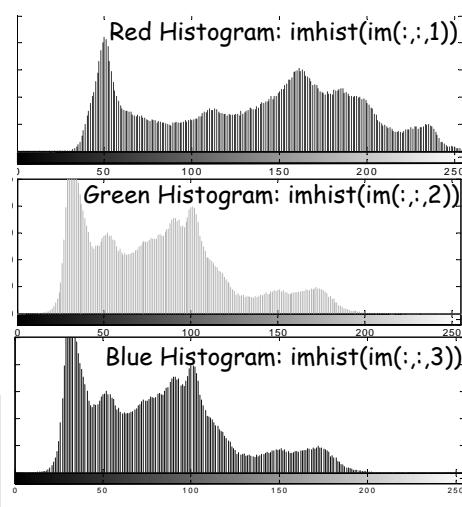
33



Which Histogram ?



Gray Histogram: `(rgb2gray)`



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Histograms

Given an array of numbers:

$$A = \left\{ \begin{array}{cccccccccc} 2 & 9 & 4 & 5 & 2 & 2 & 5 & 6 & 1 \\ 6 & 9 & 3 & 7 & 3 & 5 & 4 & 2 & 0 \\ 3 & 7 & 8 & 0 & 7 & 7 & 5 & 4 & 8 \\ 9 & 4 & 0 & 6 & 6 & 5 & 3 & 0 & 1 \\ 7 & 4 & 7 & 0 & 4 & 6 & 4 & 9 & 2 \end{array} \right\}$$

Find the frequency of each number:

Number:	0	1	2	3	4	5	6	7	8	9
Count:	5	2	5	4	7	5	5	6	2	4

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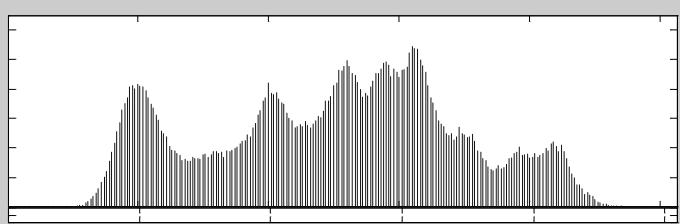
Histogram (Normal)



x image matrix (intensity)

```
imhist(x) display histogram  
[count, bin] = imhist(x);
```

Count
pixels



3

Gray level

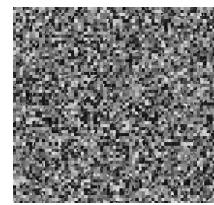
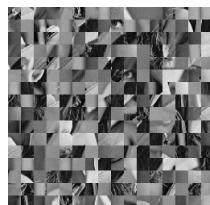
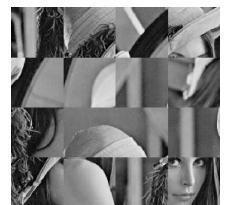


Histograms (cont.)

The histogram of an image contains valuable information concerning the distribution of gray levels

It does not contain any spatial information

All the following images have exactly the same histograms!



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Histograms as Voting

A histogram is a result of voting – it counts the number of supporters (i.e., pixels) of each candidate (graylevel)

In the simple case of a binary image there are only two candidates: Mr zero and Ms one.

Voting has many useful applications in image processing (as well as in democracy).

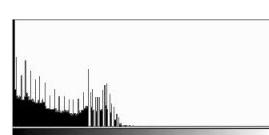
50



Image global deformations (1)



(a)



Too dark



(b)



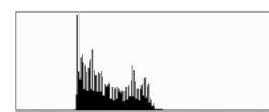
Too bright



Image global deformations (2)



(c)



Low contrast



(d)

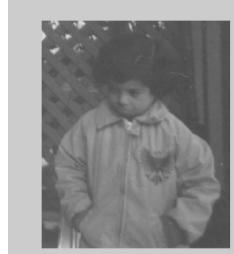


High contrast

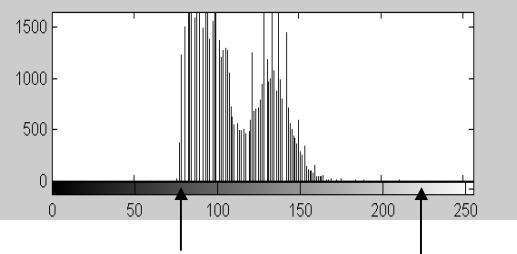


Histogram stretching

I



Histogram of I



```
I = imread('pout.tif');  
a = min(I(:));  
b = max(I(:));  
J = 255*(I-a)/(b-a);  
J = uint8(J);
```

$$J = \frac{I - I_{\min}}{I_{\max} - I_{\min}} \cdot 255$$

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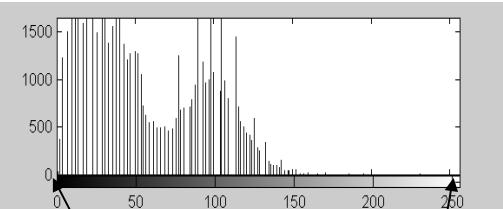


Histogram stretching (cont.)

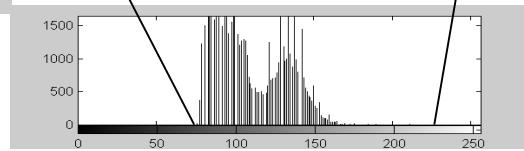
J



Histogram of J



Histogram of I



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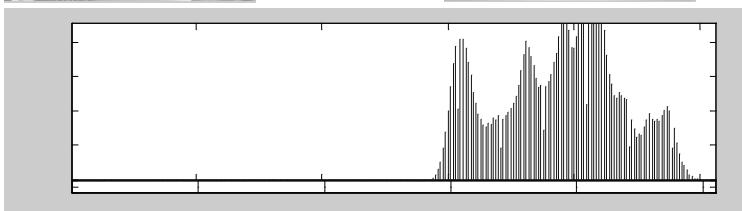
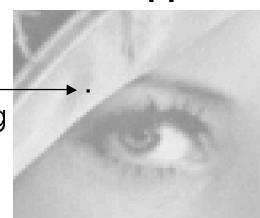


Stretching limitations

What happened?



A single pixel
Change
Plus rescaling



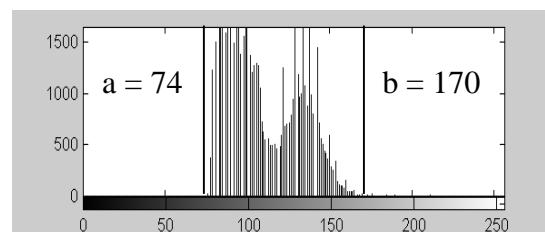
The minimum is 0 and the maximum is 255

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Histogram adjustment

Histogram of I $K = \text{imadjust}(I, [0.3 \ 0.67], [])$;



$$K = \begin{cases} 255 & \text{if } I_i \geq b \\ 255 * (I_i - a) / (b - a) & \text{if } a \leq I_i < b \\ 0 & \text{if } I_i < a \end{cases}$$

This is a non-linear operation

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Histogram adjustment (cont.)

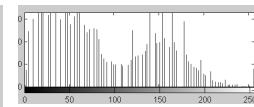
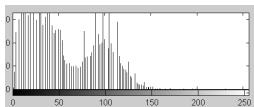
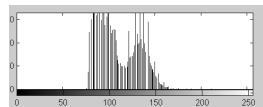
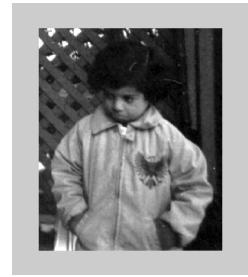
I



J



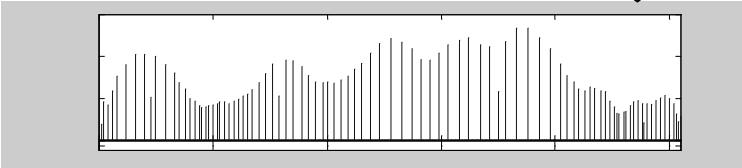
K



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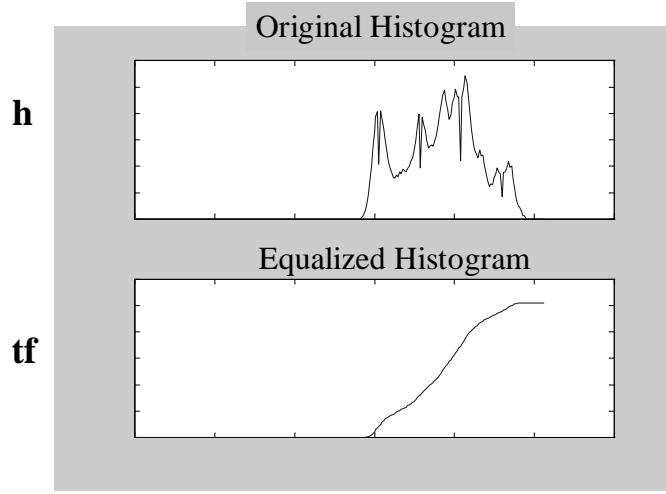
Histogram equalization

 $y = \text{histeq}(x, 256);$ 

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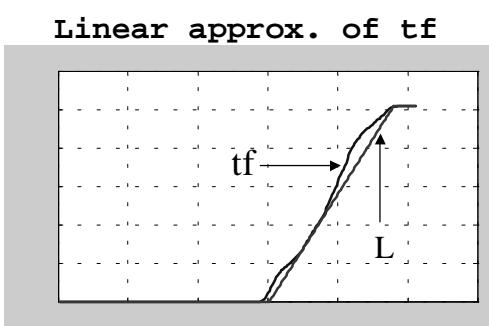
Transfer Function



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Look Up Table (LUT)

**X** (in [0 255])

```
L(1:150) = 0;  
L(151:240) = linspace(0,255,90);  
L(241:255) = 255;  
  
y = L(x+1);
```

y

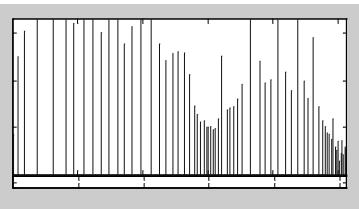
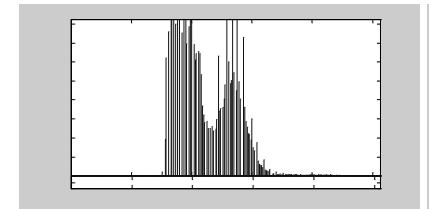
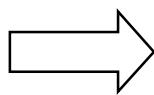


Dr. Yoram Tal



Histogram equalization (3)

Pout



Dr. Yoram Tal

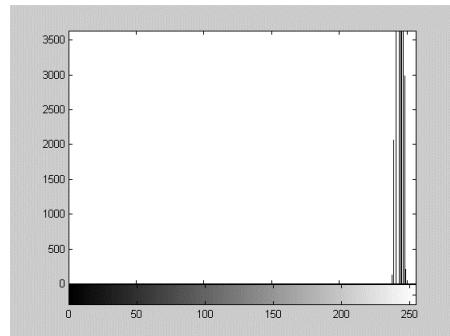


Histogram equalization (4) Determining the grayscale precision of a scanner

paper



Histogram of paper



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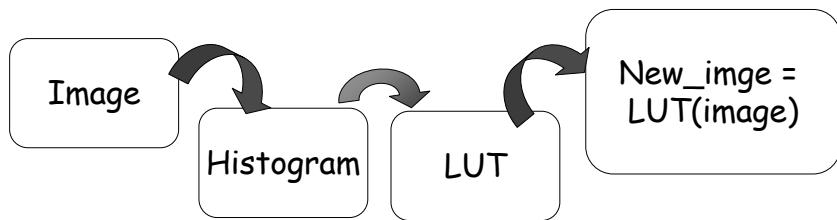
Histogram Manipulation

Why?

- Correct illumination problems
 - Correct contrast problems

How?

- Use histogram information to create a Look-Up Table (LUT)
 - Apply LUT to the image



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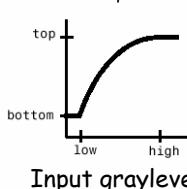
Gamma Correction

(point operation)

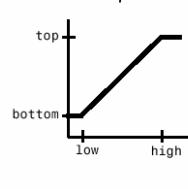
Output gray|level

brighten

$$\gamma < 1$$

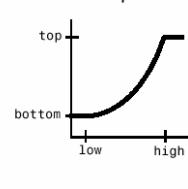


γ = 1



darken

$$\gamma > 1$$



$$I_{out} = 255 \cdot \left(\frac{I - I_{\min}}{I_{\max} - I_{\min}} \right)^{\gamma}$$

(gamma + stretching)

```
[H, b] = imhist(I);
p = H/sum(H);      (p is in [0,1])
LUT = p.^gamma
```

$$I_{out} = LUT(I+1); \quad (\text{if } I_{min} \text{ is } 0)$$

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Option I

operate on each channel separately
Warning: distortion of colors!



```
R = brighten(I(:,:,1),0.4);
G = brighten(I(:,:,2),0.4);
B = brighten(I(:,:,3),0.4);
```

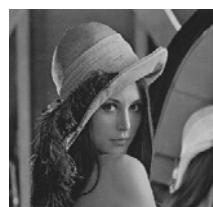
```
RGB = cat(3,R,G,B);
```

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Option II

operate on the intensity channel



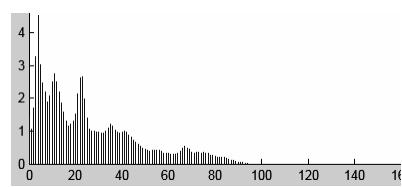
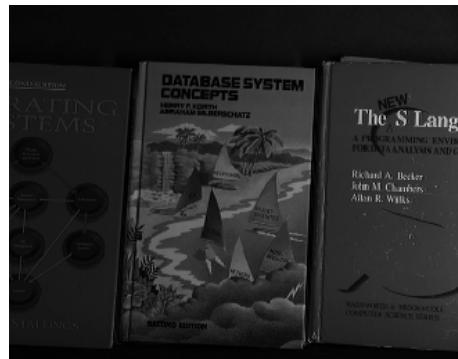
brighten

$$\gamma = \begin{cases} 1 - \beta, & \beta > 0 \\ \frac{1}{1 + \beta}, & \beta \leq 0 \end{cases}$$

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Histogram Equalization



Assumption: A “good” image has a “flat” histogram, i.e., all graylevels have the same probability.

See histeq

How ? →

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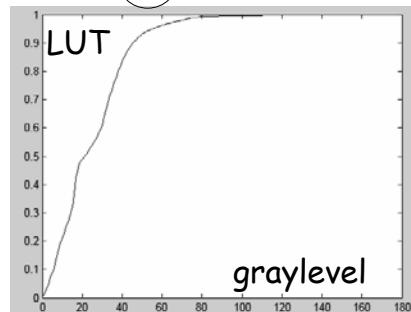
Histogram Equalization - Lemma

- Let I be an image having N pixels and $G+1$ graylevels such that g is a graylevel of I iff $g \in \{0, 1, 2, \dots, G\}$
- Let h be the histogram of I where $h(g)$ is the number of pixels in I having graylevel g .
- Let H be the cumulative histogram of I : $H(x) = \sum_{g=0}^{g=x} h(g)$
- Then $L(x) = (G/N) H(x)$ is a lookup table which transforms I into a histogram equalized image.

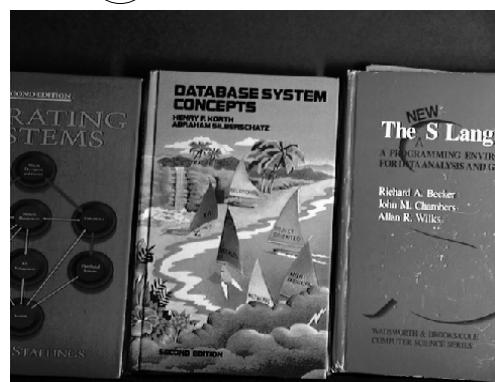
40

Histogram Equalization - how

1 Make LUT



2 Apply LUT



```
[H, b] = imhist(I);
p = cumsum(H); p = p/p(end);
LUT = p*255;
```

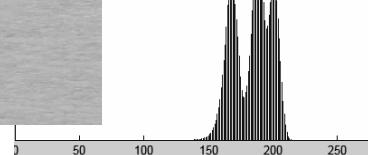
$I_{out} = LUT(I+1);$ (if I is uint8)

The same result can be achieved with histeq

In this case, a similar effect could be obtained with imadjust
(use stretchlim to find thresholds)

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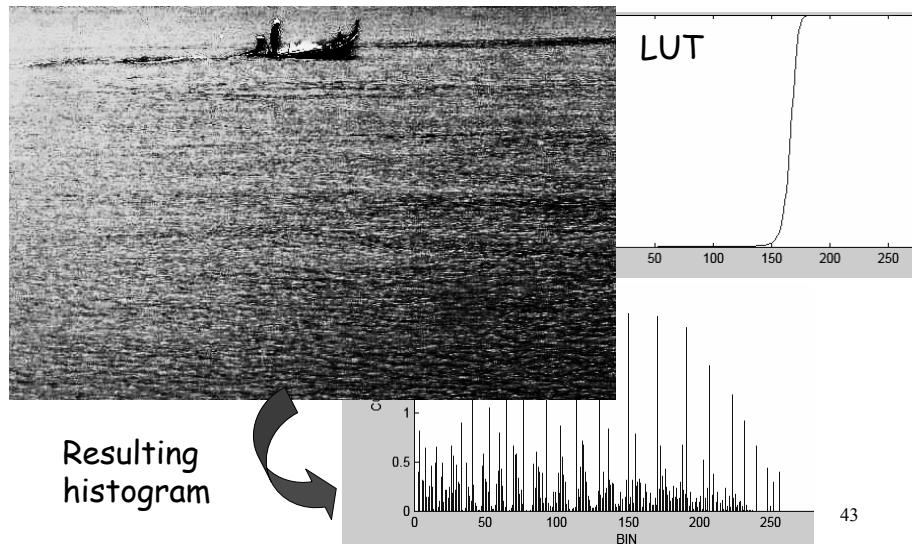
Histogram Equalization problem: a narrow histogram



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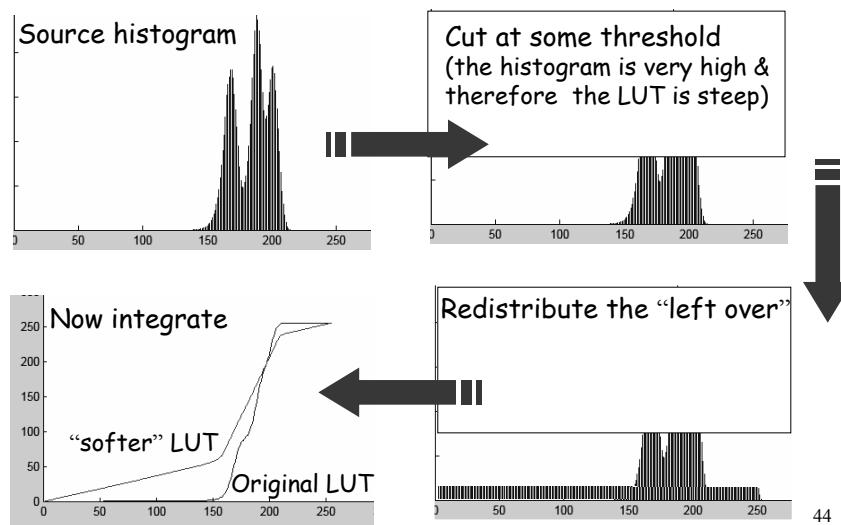
Histogram Equalization result: too much contrast



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CLHE Contrast Limited Histogram Equalization



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CLHE - Result



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CLAHE

Contrast Limited Adaptive Histogram equalization

Problem: a big illumination difference between the two sides of the image (High Dynamic Range).



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Try Histogram Equalization

It certainly helps, but does not solve the main problem.



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Adaptive Histogram Equalization

```
rgb => hsv;
v1 = adapthisteq(v,{parameters});
hsv1 => rgb1
```



- The image is partitioned into blocks.
- Each block is enhanced with CLHE.
- The results are stitched back smoothly.

“Parameters” are very important!

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CLAHE – another example

Source image



Histogram Eq.



adapthisteq



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