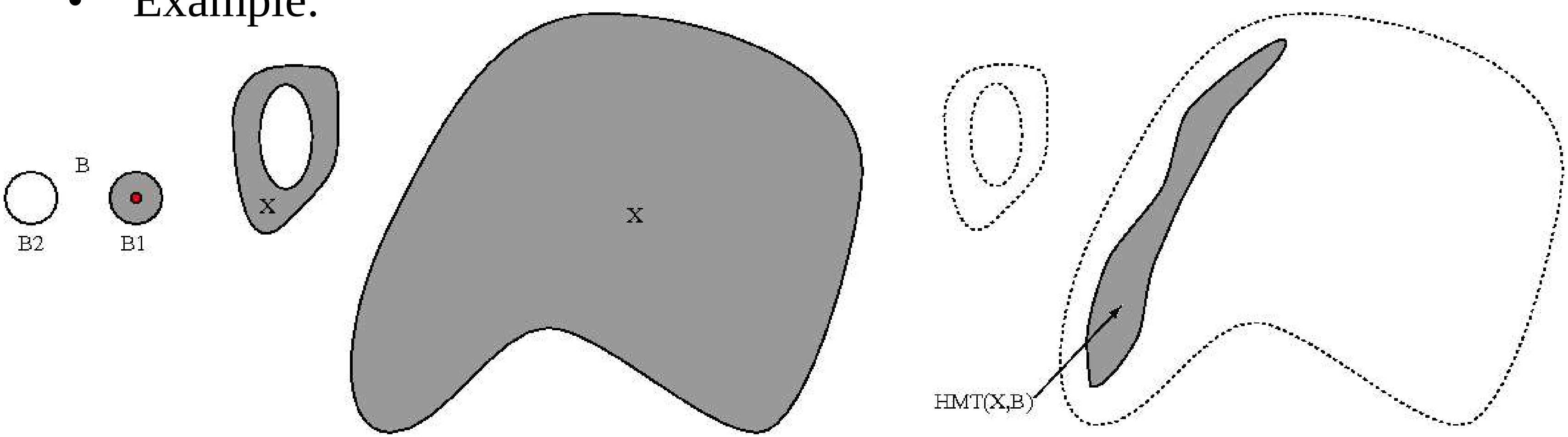
Hit-or-miss transform

- Used to extract pixels with specific neighbourhood configurations from an image
- Grey scale extension exist
- Uses two structure elements B1 and B2 to find a given foreground and background configuration, respectively

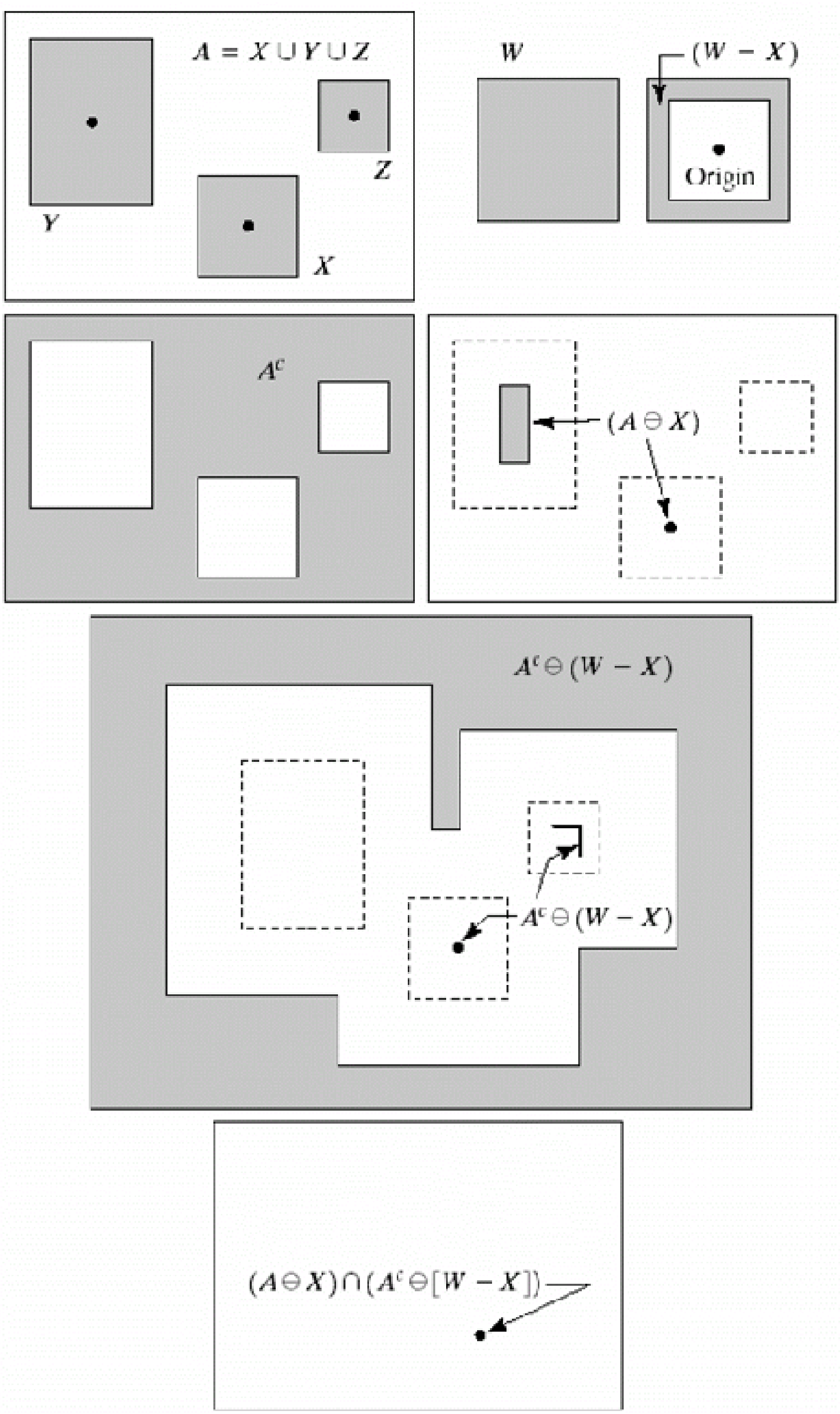
$$HMT_B(X) = \{ x \mid (B_1)_x \subseteq X, (B_2)_x \subseteq X^C \}$$

- Example:



9.4 The hit-or-miss transformation

Illustration...



- Objective is to find a disjoint region (set) in an image

- If B denotes the set composed of X and its background, the match/hit (or set of matches/hits) of B in A, is

$$A \circledast B = (A \ominus X) \cap [A^c \ominus (W - X)]$$

- Generalized notation: $B = (B_1, B_2)$

  - B_1: Set formed from elements of B associated with an object

  - B_2: Set formed from elements of B associated with the corresponding background

  [Preceeding discussion: $B_1 = X$ and $B_2 = (W - X)$]

  - More general definition:

$$A \circledast B = (A \ominus B_1) \cap [A^c \ominus B_2]$$

  - $A \circledast B$ contains all the origin points at which, simultaneously, B_1 found a hit in A and B_2 found a hit in A^c

Hit-or-miss transform

$$HMT_B(X) = \{x \mid (B_1)_x \subseteq X, (B_2)_x \subseteq X^C\}$$

- Can be written in terms of an intersection of two erosions:

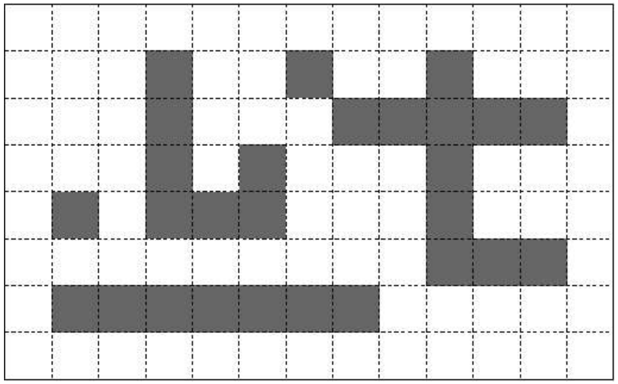
$$HMT_B(X) = \varepsilon_{B_1}(X) \cap \varepsilon_{B_2}(X^c)$$

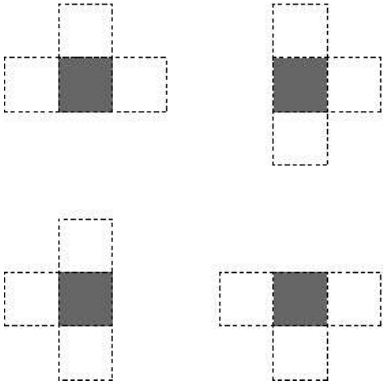
Hit-or-miss transform

- Simple example usages - locate:
  - Isolated foreground pixels
    - no neighbouring foreground pixels
  - Foreground endpoints
    - one or zero neighbouring foreground pixels
  - Multiple foreground points
    - pixels having more than two neighbouring foreground pixels
  - Foreground contour points
    - pixels having at least one neighbouring background pixel

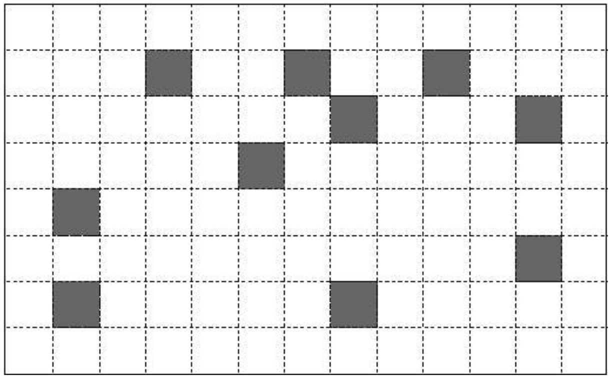
Hit-or-miss transform example

- Locating 4-connected endpoints





SEs for 4-connected endpoints



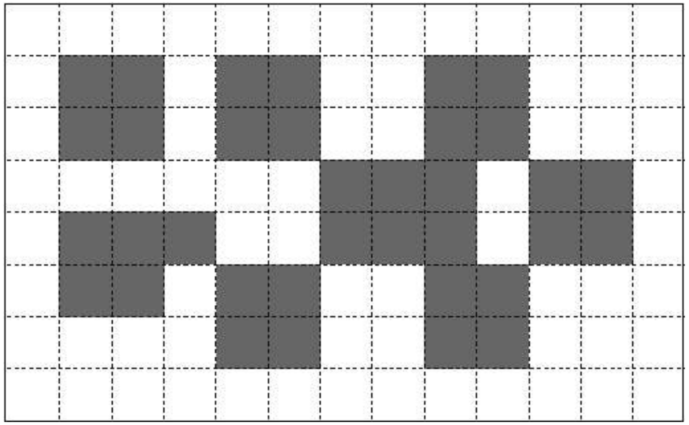
Resulting Hit-or-miss transform

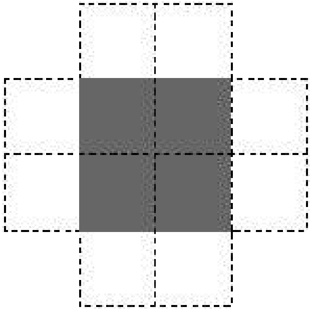
Hit-or-miss opening

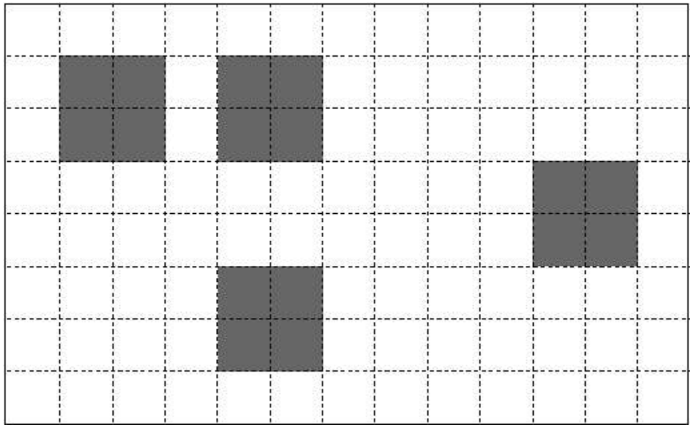
- Objective: keep all points that fit the SE.
- Definition:

$$\tilde{\gamma}_B(X) = \delta_{\check{B}_1} HMT_B(X) = \delta_{\check{B}_1} \varepsilon_{B_1}(X) \cap \varepsilon_{B_2}(X^c)$$

hit-or-miss opening example







Resulting hit-or-miss opening

• Alternative definition:

$$A \circledast B = (A \ominus B_1) - (A \oplus \hat{B}_2)$$

• A background is necessary to detect disjoint sets

• When we only aim to detect certain patterns within a set, a background is not required, and simple erosion is sufficient

9.5 Some basic morphological algorithms

When dealing with **binary images**, the principle application of morphology is extracting image components that are useful in the representation and description of shape

9.5.1 Boundary extraction

The boundary $\beta(A)$ of a set A is

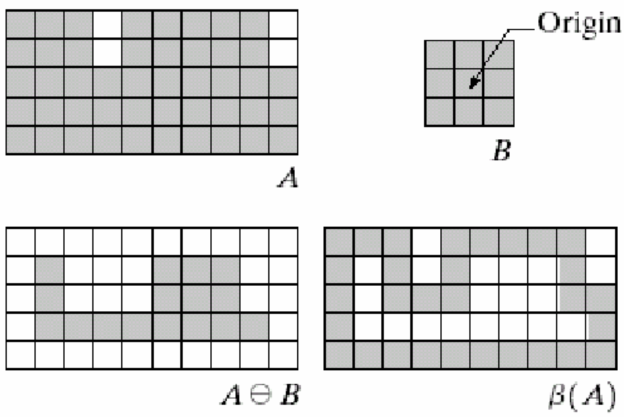
$$\beta(A) = A - (A \ominus B),$$

where B is a suitable structuring element

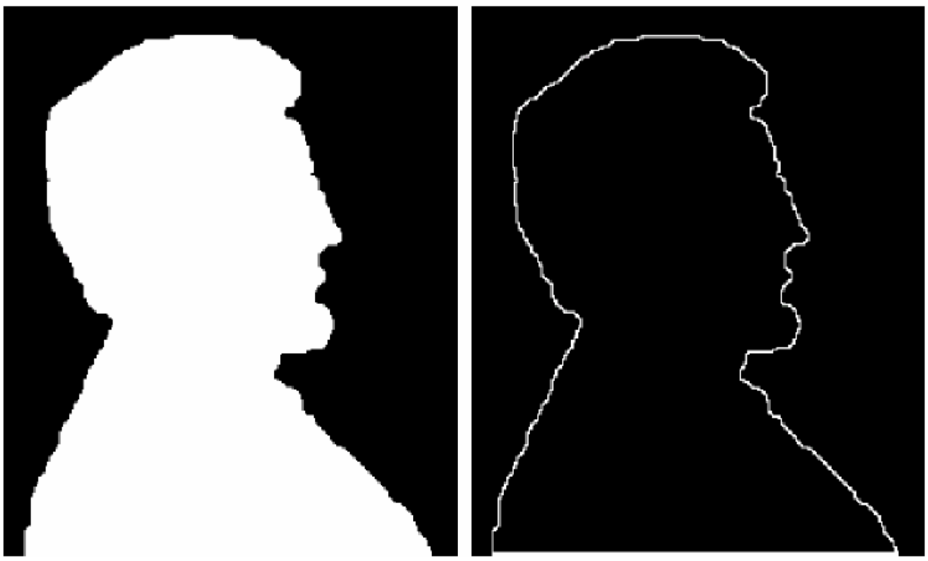
Illustration...

a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



Example 9.5: Morphological boundary extraction



a b

FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

9.5.2 Region filling

• Begin with a point p inside the boundary, and then fill the entire region with 1's

• All non-boundary (background) points are labeled 0

• Assign a value of 1 to p to begin...

• The following procedure fills the region with 1's,

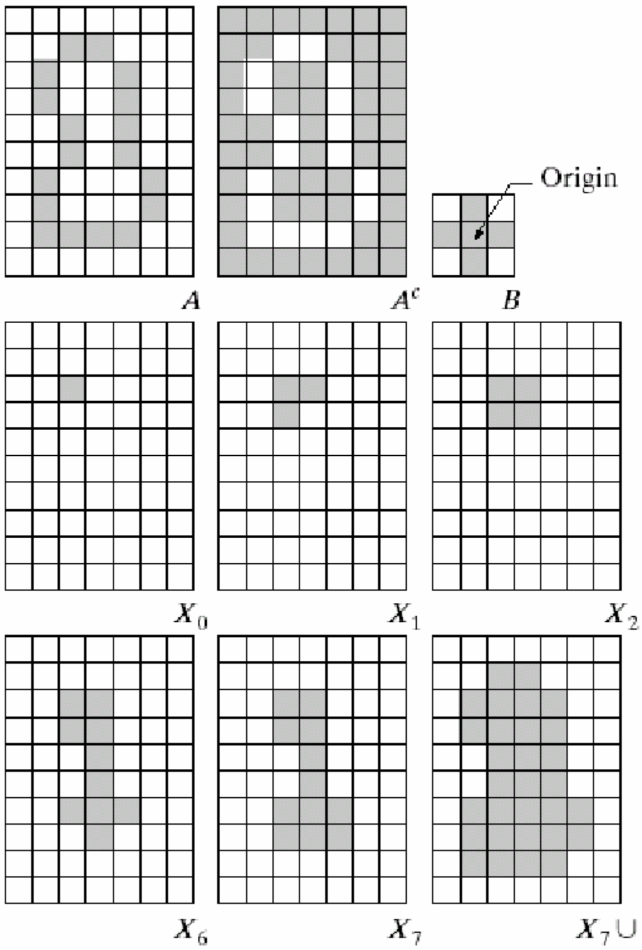
$$X_k = (X_{k-1} \oplus B) \cap A^c, \quad k = 1, 2, 3, \ldots,$$

where $X_0 = p$, and B is the symmetric structuring element in figure 9.15 (c)

• The algorithm terminates at iteration step k if $X_k = X_{k-1}$

• The set union of X_k and A contains the filled set and its boundary

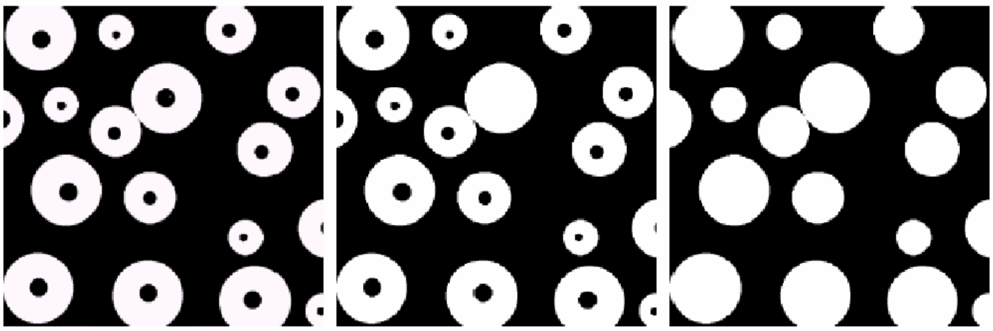
Note that the intersection at each step with A^c limits the dilation result to inside the region of interest



a b c
d e f
g h i

FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

Example 9.6: Morphological region filling



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

9.5.3 Extraction of connected components

Let Y represent a connected component contained in a set A and assume that a point p of Y is known. Then the following iterative expression yields all the elements of Y:

$$X_k = (X_{k-1} \oplus B) \cap A \quad k = 1, 2, 3, \ldots,$$

where $X_0 = p$, and B is a suitable structuring element. If $X_k = X_{k-1}$, the algorithm has converged and we let $Y = X_k$.

This algorithm is applicable to any finite number of sets of connected components contained in A, assuming that a point is known in **each** connected component

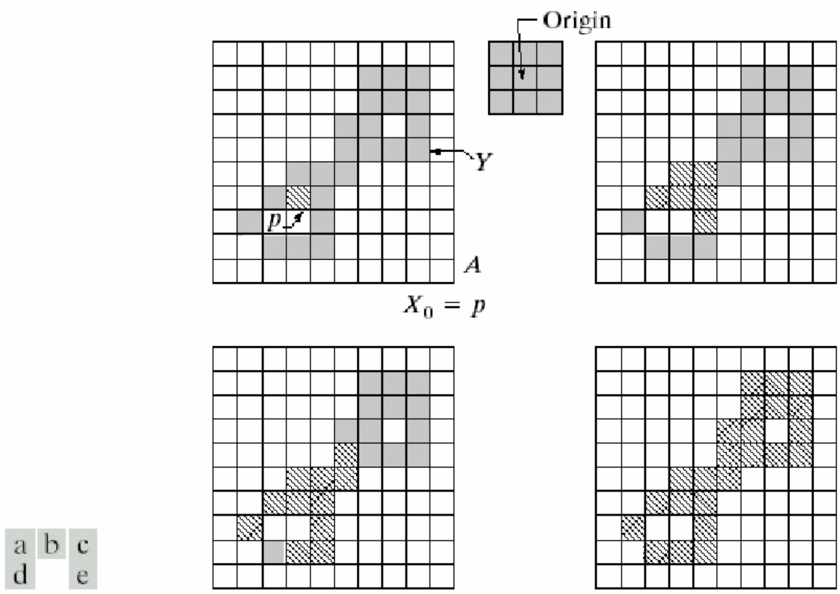


FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

Example 9.7:

a
b
c d

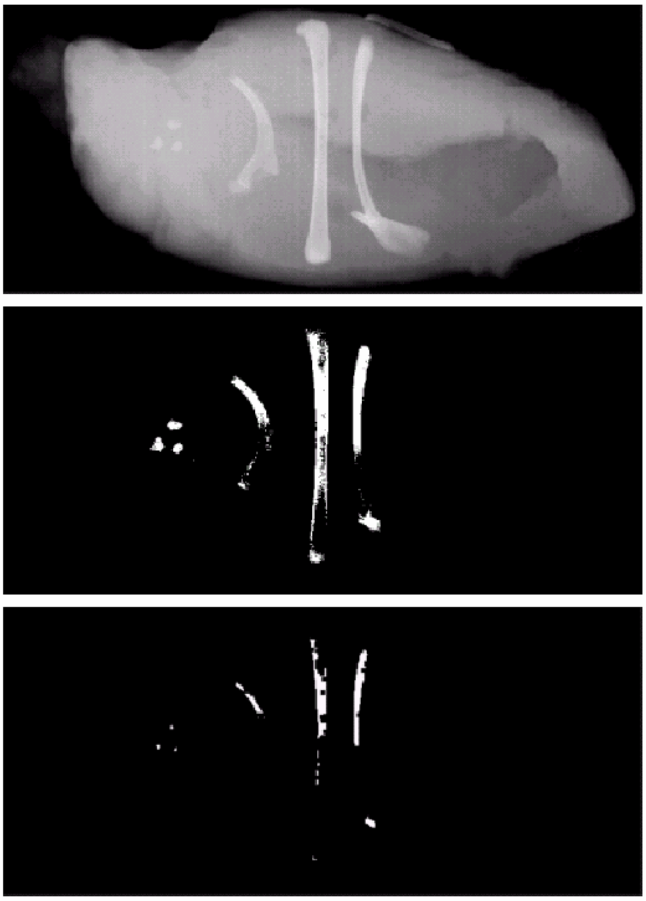


FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

9.5.4 Convex hull

Morphological algorithm for obtaining the convex hull, $C(A)$, of a set A...

Let B_1, B_2, B_3 and B_4 represent the four structuring elements in figure 9.19 (a), and then implement the equation ...

$$X_k^i = (X_{k-1} \circledast B^i) \cup A,\ i = 1, 2, 3, 4,\ k = 1, 2, \ldots,\ X_0^i = A$$

Now let $D^i = X^i_{\text{conv}}$, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. Then the convex hull of A is

$$C(A) = \cup_{i=1}^4 D^i$$

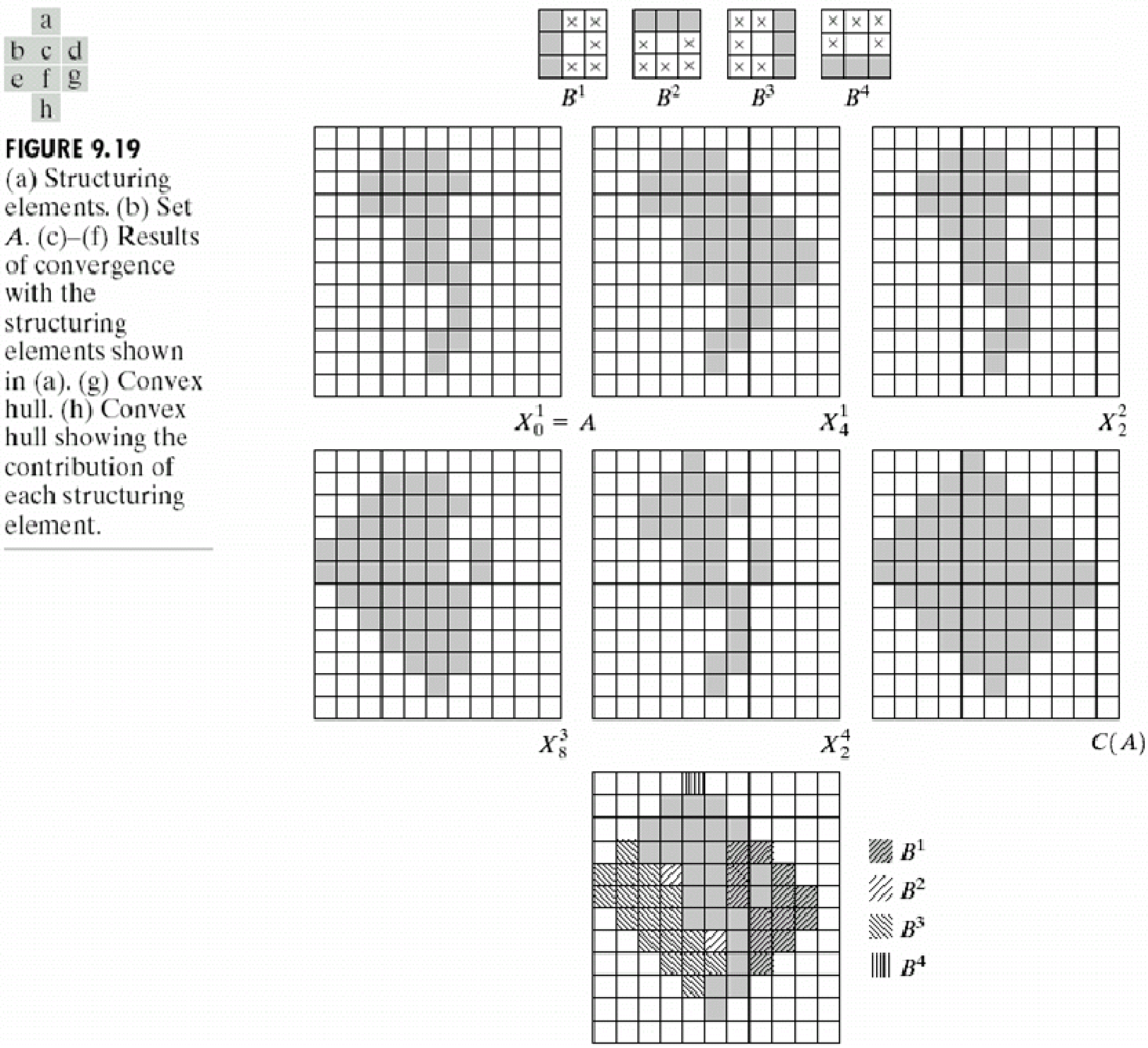


FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

Shortcoming of above algorithm: convex hull can grow beyond the minimum dimensions required to guarantee convexity

Possible solution: Limit growth so that it does not extend past the vertical and horizontal dimensions of the original set of points

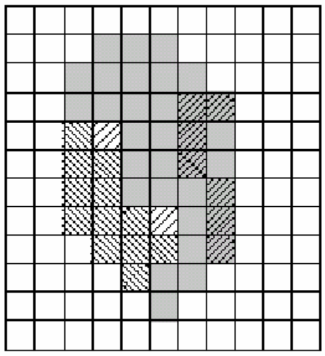


FIGURE 9.20 Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

Boundaries of greater complexity can be used to limit growth even further in images with more detail

9.5.5 Thinning

The thinning of a set A by a structuring element B:

$$A \otimes B = A - (A \circledast B) = A \cap (A \circledast B)^c$$

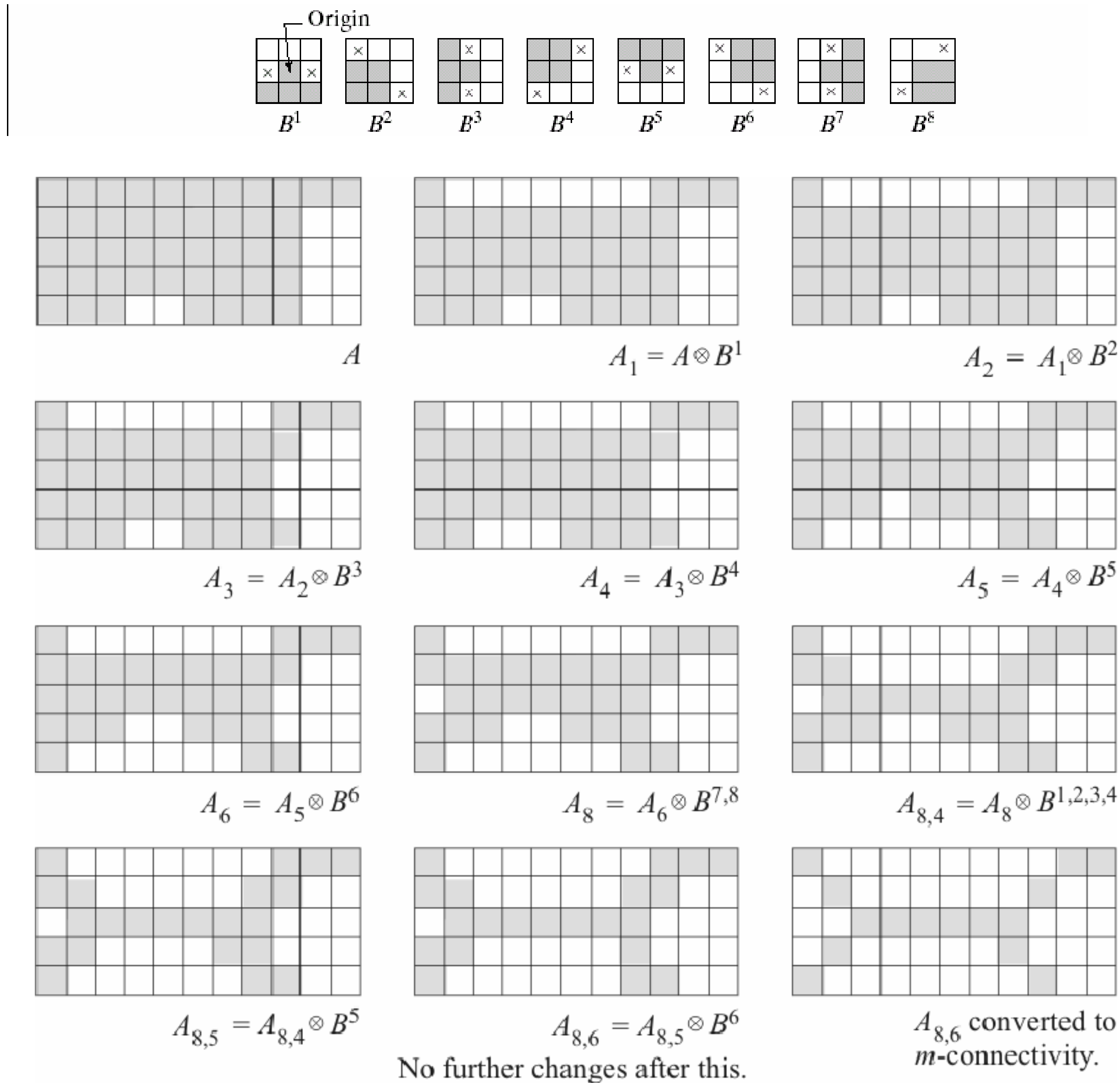
Symmetric thinning: sequence of structuring elements,

$$\{B\} = \left\{B^1, B^2, B^3, \ldots, B^n\right\},$$

where B^i is a rotated version of B^{i-1}

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$

Illustration: Note that figure 9.21 (in the handbook) has many errors – this one is correct...

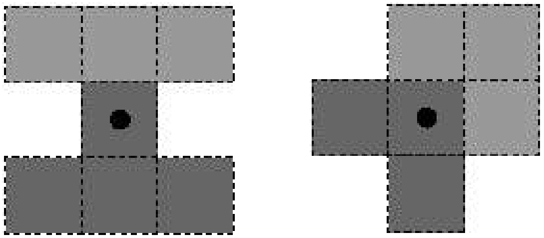


Thinning

- Used to shrink objects in binary images
- Differs from erosion in that objects are never completely removed
  - Preserving the homotopy is often an objective
- Successive thinning until stability results in object skeletons
- Thinning is defined as:

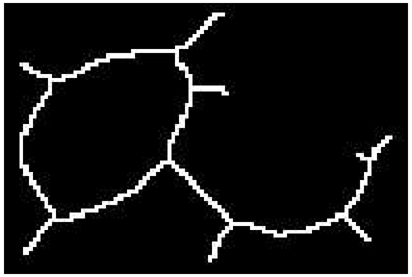
$$THIN(X, B) = X \setminus HMT_B(X)$$

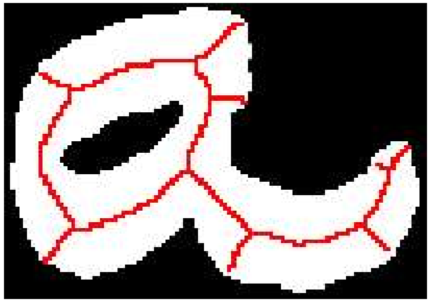
- Example structure elements used in thinning (rotated 90 degrees 3 times to create 8 structure elements):



Skeletons

- Compact or minimal representation of objects in an image while retaining homotopy of the image
- As stated earlier, the skeletons of objects in an image can be found by successive thinning until stability
- The thinning cannot be executed in parallel since this may cause the homotopy of the image to change
- Example:



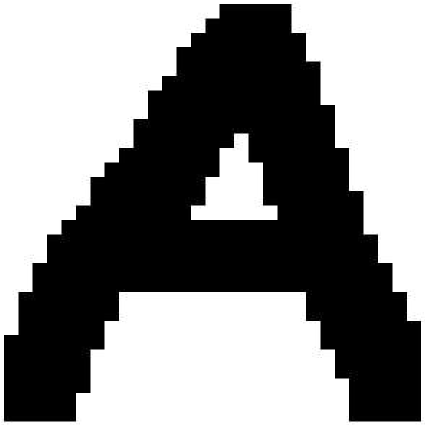


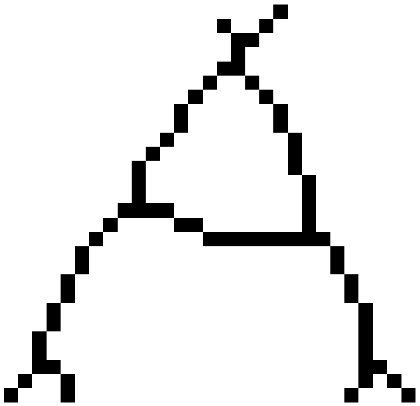
Skeleton

- The skeleton of an object is often defined as the medial axis of that object.
  - Pixels are then defined to be skeleton pixels if they have more than one “closest neighbours”.
- Some skeleton algorithms are based on this definition and are computed through the distance transform
- Other algorithms produce skeletons that are smaller than the defined medial axis (such as minimal skeletons)

Skeletons

- Problem: Finding a minimal representation
  - Solution 1: Pruning of smaller branches
    - Can use HMT to locate and remove endpoints successively

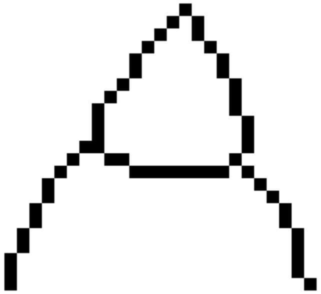




Skeleton with unwanted branches

Skeletons

- Pruning is dependent on parameter choices (maximum branch length of branches to be removed)
- Solution 2: Skeleton algorithm producing minimal skeleton
  - One such algorithm is described in [Thinning Methodologies-A Comprehensive Survey," IEEE TrPAMI, vol. 14, no. 9, pp. 869-885, 1992.]
  - HMT is not used in this algorithm



Minimal skeleton

Skeletons

- Some skeletons can be used to reconstruct the original objects in an image.

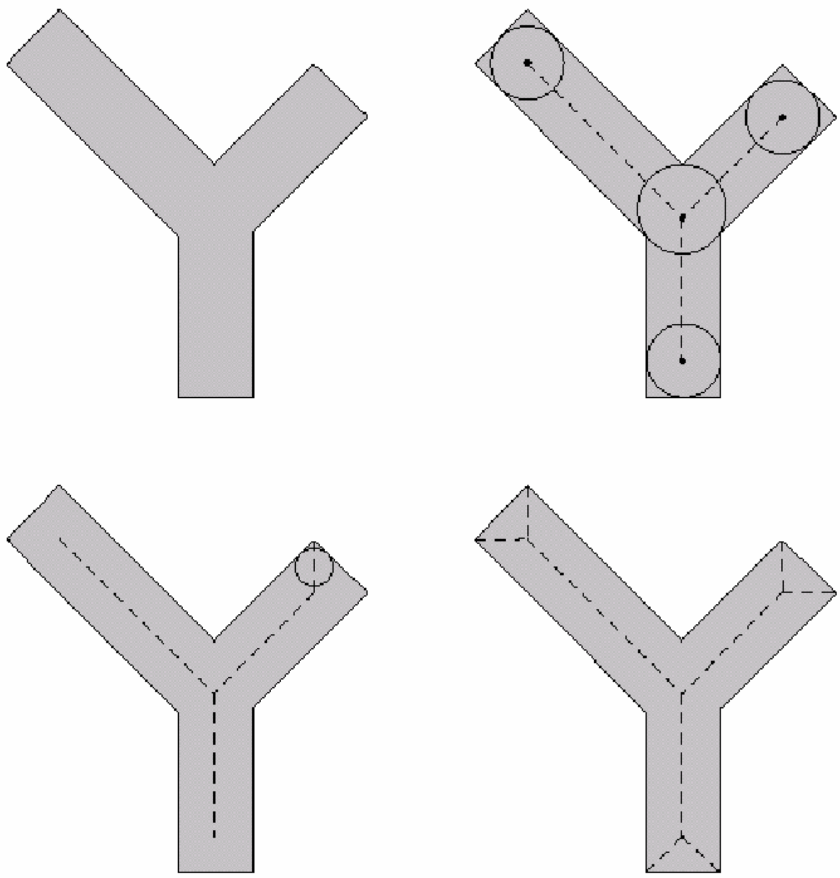
```
a=mmneg(mmframe(mmbinary(ones(7,9))))
Warning: converting image from double to int32
a =
     0     0     0     0     0     0     0     0     0
     0     1     1     1     1     1     1     1     0
     0     1     1     1     1     1     1     1     0
     0     1     1     1     1     1     1     1     0
     0     1     1     1     1     1     1     1     0
     0     1     1     1     1     1     1     1     0
     0     0     0     0     0     0     0     0     0
b=mmskelm(a,mmsecross,'value')
b =
     0     0     0     0     0     0     0     0     0
     0     1     0     0     0     0     0     1     0
     0     0     2     0     0     0     2     0     0
     0     0     0     3     3     3     0     0     0
     0     0     2     0     0     0     2     0     0
     0     1     0     0     0     0     0     1     0
     0     0     0     0     0     0     0     0     0
c=mmskelmrec(b,mmsecross)
c =
     0     0     0     0     0     0     0     0     0
     0     1     1     1     1     1     1     1     0
     0     1     1     1     1     1     1     1     0
     0     1     1     1     1     1     1     1     0
     0     1     1     1     1     1     1     1     0
     0     1     1     1     1     1     1     1     0
     0     0     0     0     0     0     0     0     0
```

Example taken from SDC Morphology Toolbox for MATLAB (www.mmorph.com)

9.5.7 Skeletons

The algorithm proposed in this section is similar to the medial axis transformation (MAT). The MAT transformation is discussed in section 11.1.5 and is far inferior to the skeletonization algorithm introduced in section 11.1.5. The skeletonization algorithm proposed in this section also does not guarantee connectivity. We therefore do not discuss this algorithm.

Illustration of the above comments...



a b
c d

FIGURE 9.23 (a) Set A. (b) Various positions of maximum disks with centers on the skeleton of A. (c) Another maximum disk on a different segment of the skeleton of A. (d) Complete skeleton.

A further illustration...

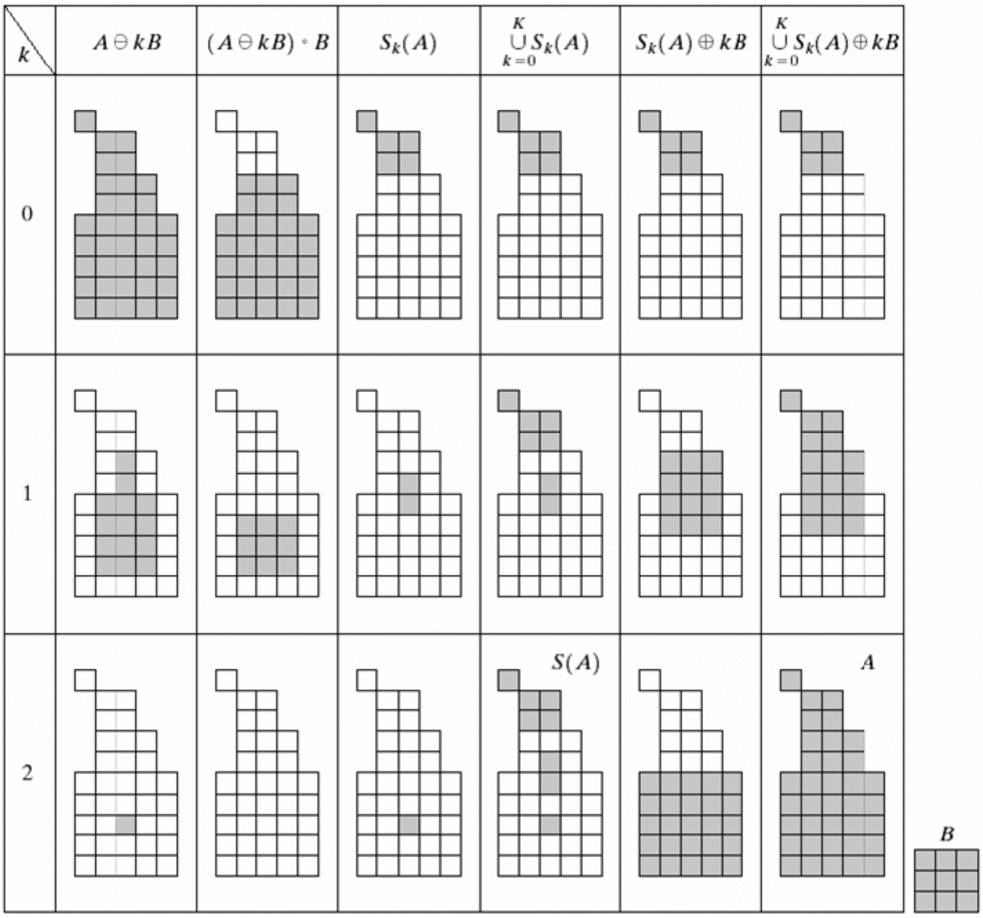


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

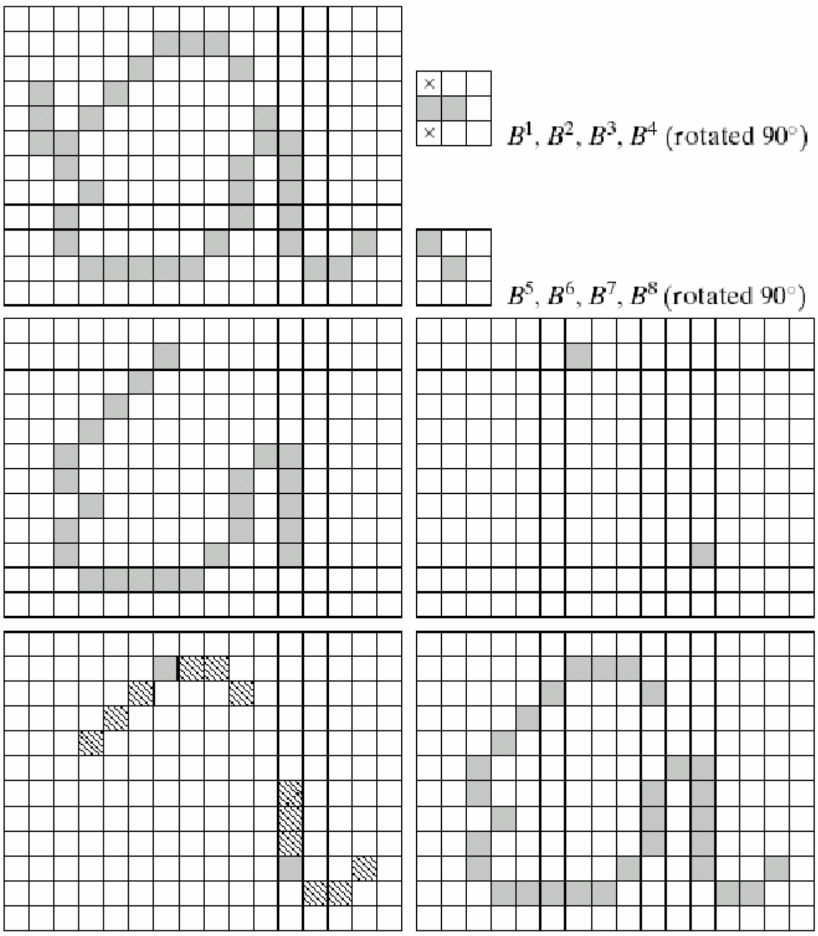
9.5.8 Pruning

- Cleans up "parasitic" components left by thinning and skeletonization

- Use combination of morphological techniques

Illustrative problem: hand-printed character recognition

- Analyze shape of skeleton of character
- Skeletons characterized by spurs ("parasitic" components)
- Spurs caused during erosion of non-uniformities in strokes
- We assume that the length of a parasitic component does not exceed a specified number of pixels



a b
c
d e
f g

FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

Any branch with three or less pixels is to be eliminated

(1) Three iterations of:

$$X_1 = A \otimes \{B\}$$

(2) Find all the end points in X_1:

$$X_2 = \cup_{k=1}^{8} \left(X_1 \circledast B^k \right)$$

(3) Dilate end points three times, using A as a delimiter:

$$X_3 = (X_2 \oplus H) \cap A, \quad H = \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

(4) Finally:

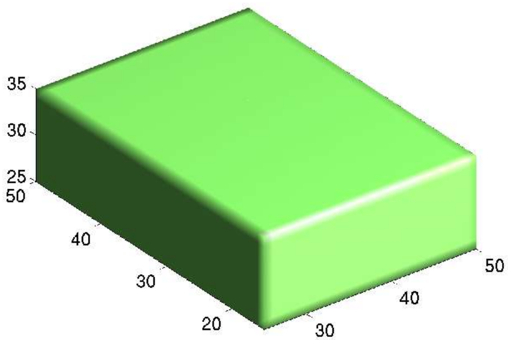
$$X_4 = X_1 \cup X_3$$

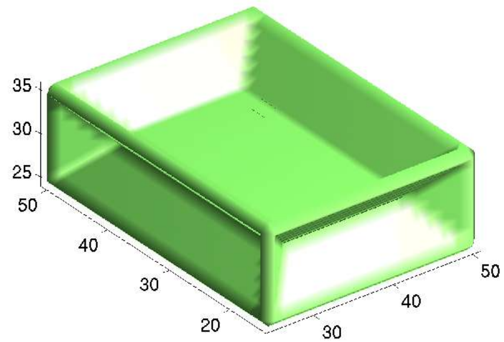
Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2) = (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Finds a connected component Y in A, given a point p in Y. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$.	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)

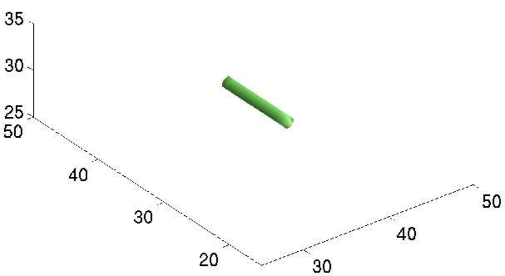
Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed.
Skeletons	$S(A) = \bigcup_{k=0} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB)$ $- [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosion of A by B. (I)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.

3D skeletons

- 3D skeletons can be divided into two groups: medial surfaces and medial lines:

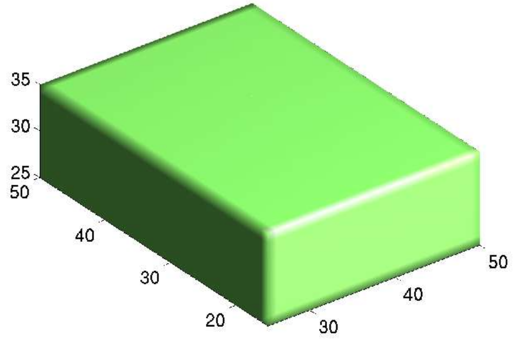


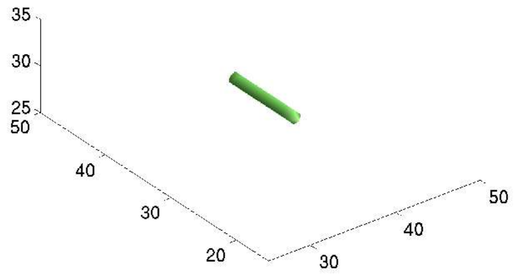




3D skeletons

- Not all 2D skeleton algorithms can be directly extended into 3D
  - Such as the example skeleton algorithm producing a minimal skeleton
- 3D skeletonization is a difficult task and much research effort has been put into this field
- A promising definition of 3D simple points is given in [Malandain, G. and Bertrand, G. (1992). Fast characterization of 3d simple points. In 11th IEEE International Conference on Pattern Recognition, pages 232–235.]





Thickening

- Thickening consists of adding border pixels instead of removing them:

$$THICK(X,B) = X \cup HMT_B(X)$$

- Thickening and thinning are dual operators:

$$THIN(X,B) = C\,THICK(X,B^c)\,C$$

9.5.6 Thickening

Thickening is the morphological dual of thinning and is defined by

$$A \odot B = A \cup (A \circledast B),$$

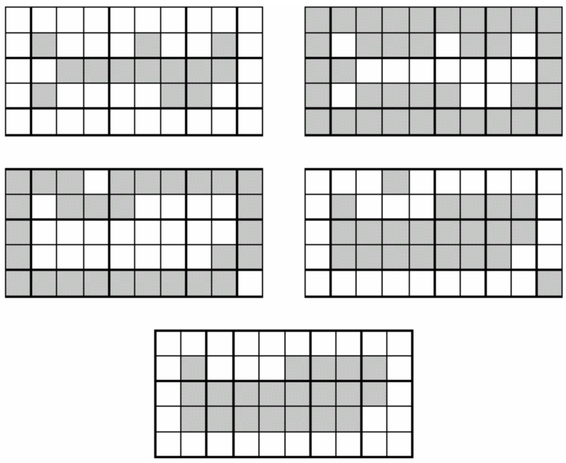
where B is a structuring element

Similar to thinning...

$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2) \ldots) \odot B^n)$$

Structuring elements for thickening are similar to those of figure 9.21 (a), but with all 1's and 0's interchanged

A separate algorithm for thickening is seldom used in practice — we thin the background instead, and then complement the result

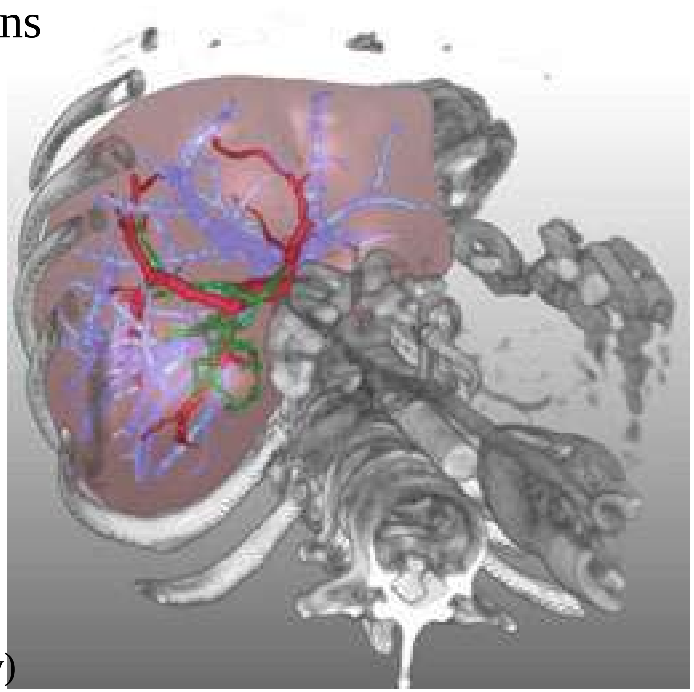


a b
c d
e

FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Practical example – reconstruction of liver vessels from CT or MR scans

- The liver is largest internal organ in the human body
- Has several functions:
  - Secretes bile
  - Produces glycogen
  - Assimilates carbohydrates, fats, and proteins
  - Manufactures essential blood components
  - Filters harmful substances from the blood

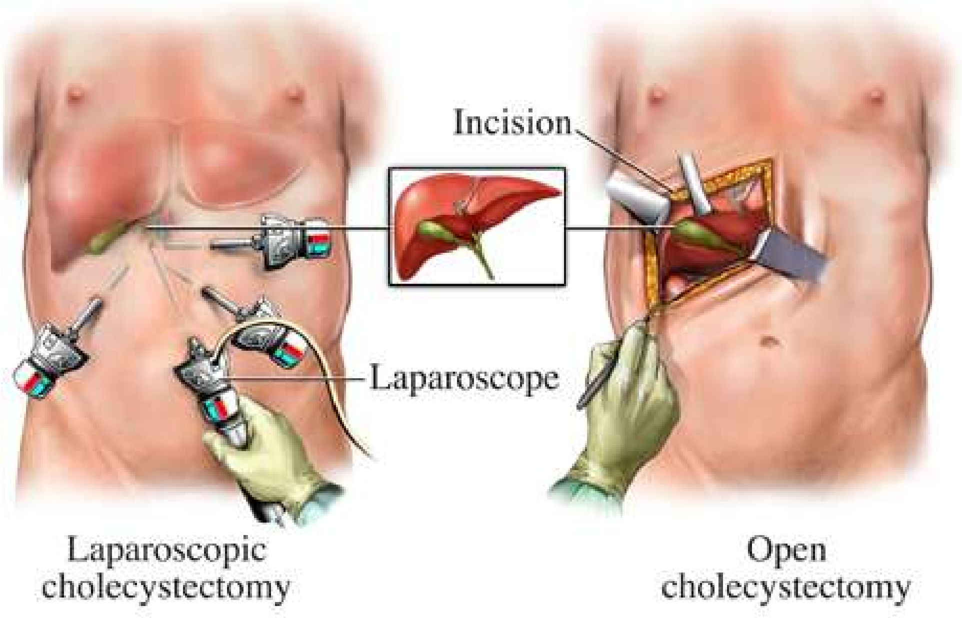


(MeVis, Preoperative Planning in Liver Surgery)

Tumours in the liver

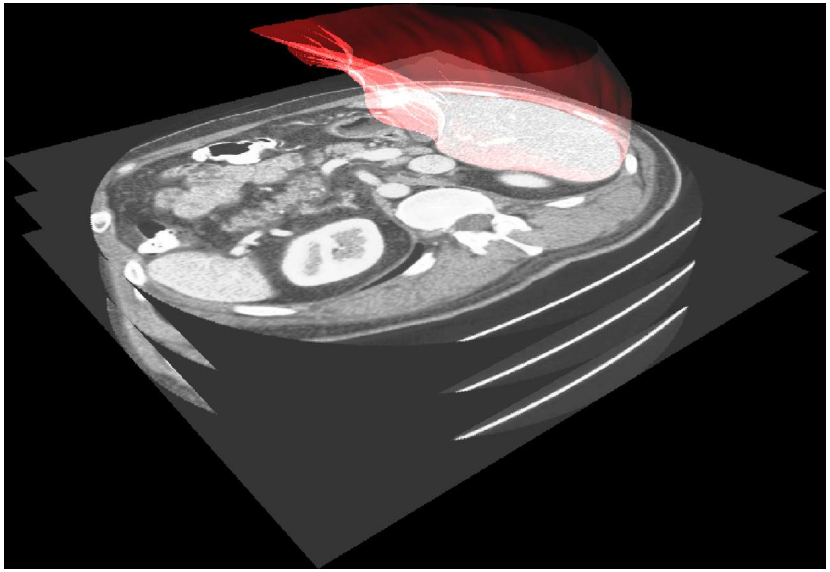
- Considered a serious medical complication
- Three ways to remove the tumours
  - Chemotherapy
  - Open surgery
  - Laparoscopic surgery (minimally invasive surgery)

(www.nucleusinc.com)



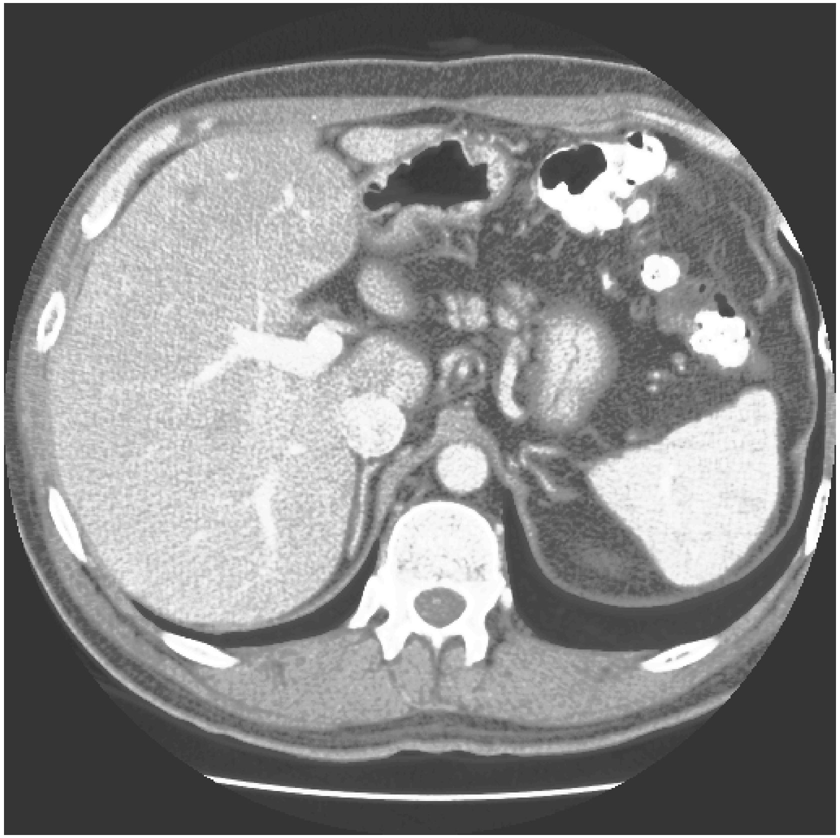
Planning liver surgery

- CT or MR scans are studied prior to a liver surgery
  - Blood vessels are tracked, and tumours are identified and located
    - Important with respect to the choice of surgical procedure
    - **Problem: Human interpretation is time-consuming and error-prone**
    - **Our goal: To perform this analysis automatically**



Automatic modelling of liver anatomy

- Step 1 – Segment the liver

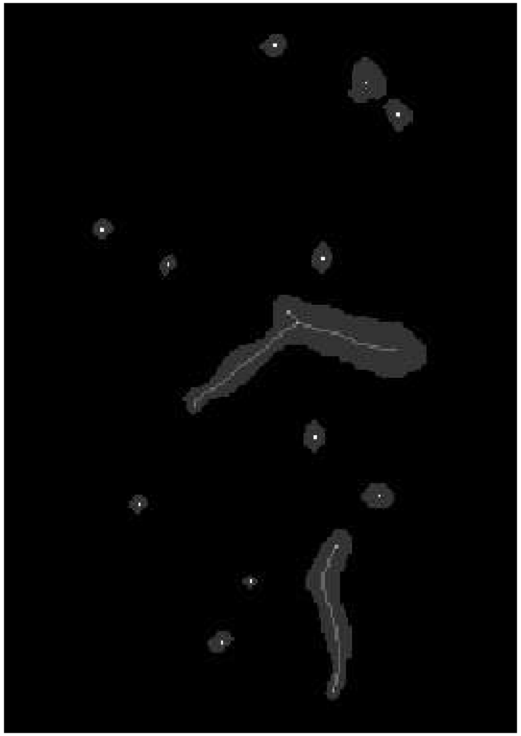


Automatic modelling of liver anatomy

- Step 2 – Segment the liver vessels

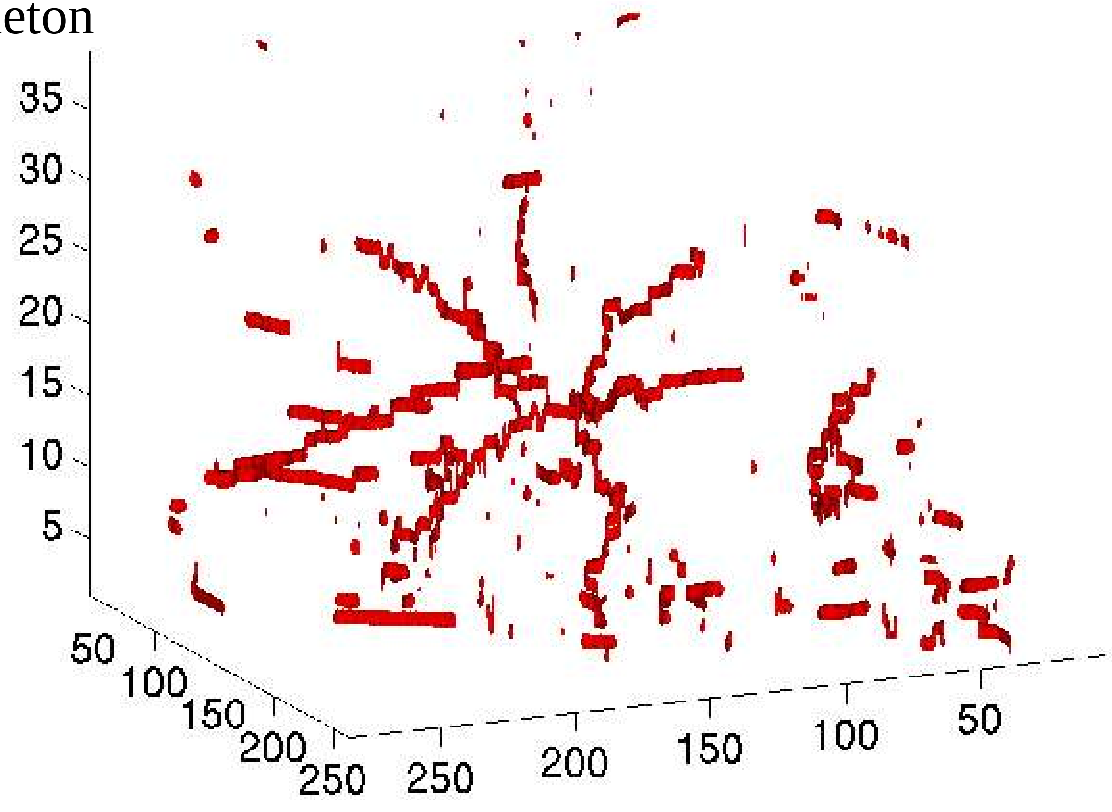
Automatic modelling of liver anatomy

- Step 3 – Identify the vessel paths



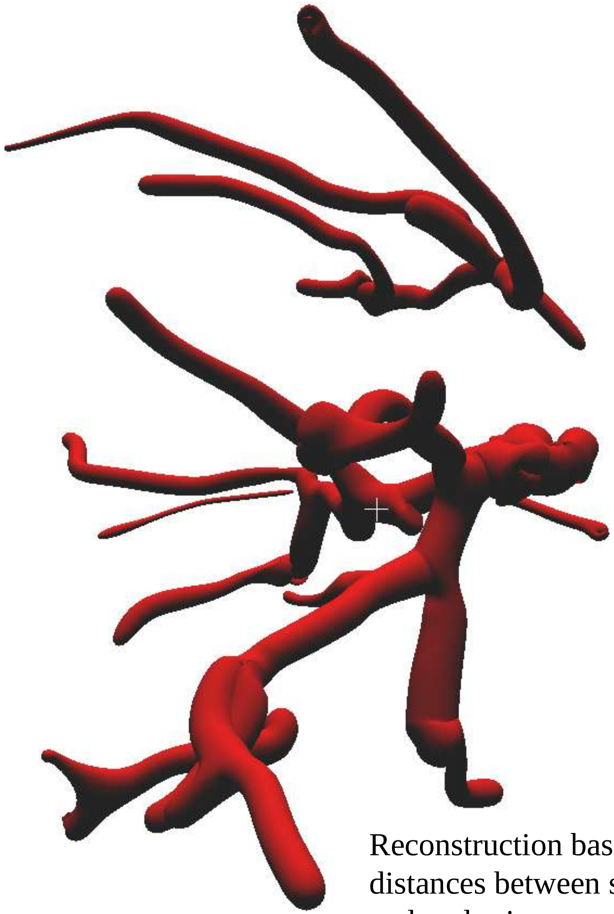
Automatic modelling of liver anatomy

- Resulting 3D skeleton
- A vessel graph (nodes and interconnections) is then extracted from this skeleton

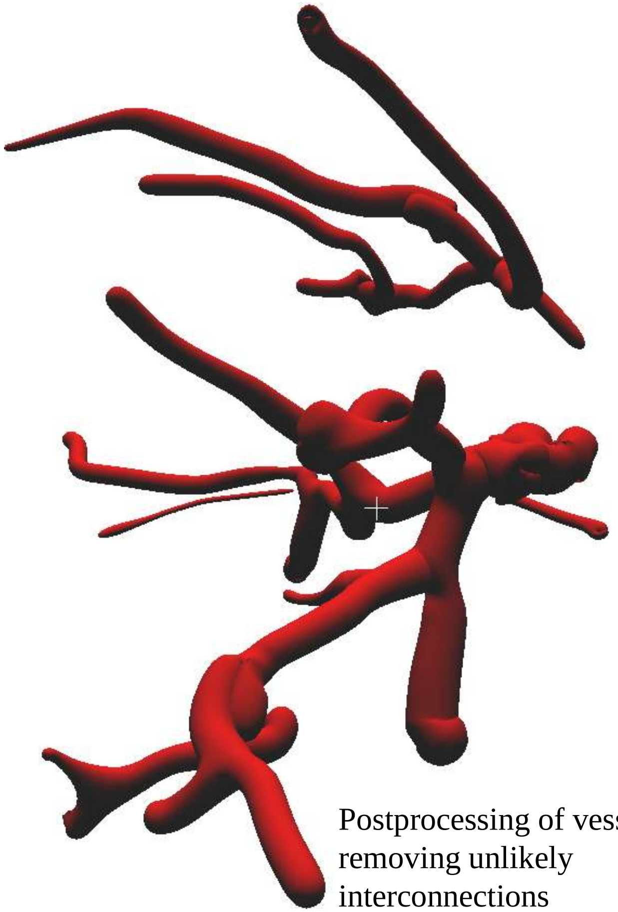


Automatic modelling of liver anatomy

- Resulting reconstruction and visualisation



Reconstruction based on the skeletons, distances between skeleton structures, and node sizes



Postprocessing of vessel graph removing unlikely interconnections

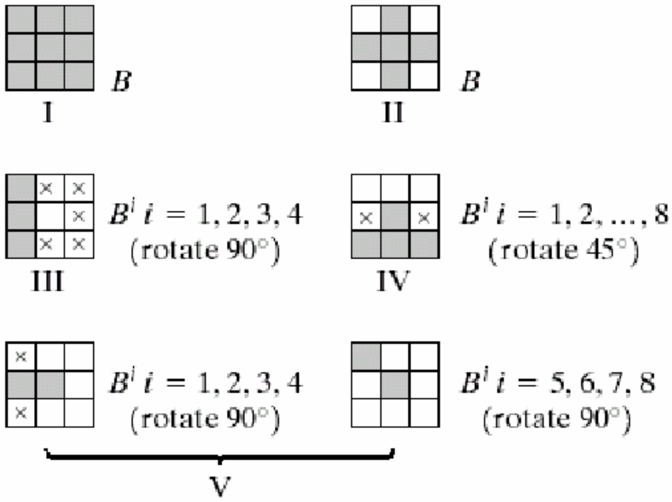


FIGURE 9.26 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

9.6 Extensions to grey-scale images

$f(x, y)$: Input image
$b(x, y)$: Structuring element image

9.6.1 Dilation

Grey-scale dilation of f by b, is defined as

$$(f \oplus b)(s,t) = \max\{f(s-x, t-y) + b(x,y) \\ |(s-x),(t-y) \in D_f;\ (x,y) \in D_b\},$$

where D_f and D_b are the domains of f and b, respectively

Simple 1D example. For functions of one variable:

$$(f \oplus b)(s) = \max \{ f(s-x) + b(x) \\ | (s-x) \in D_f;\, x \in D_b \}$$

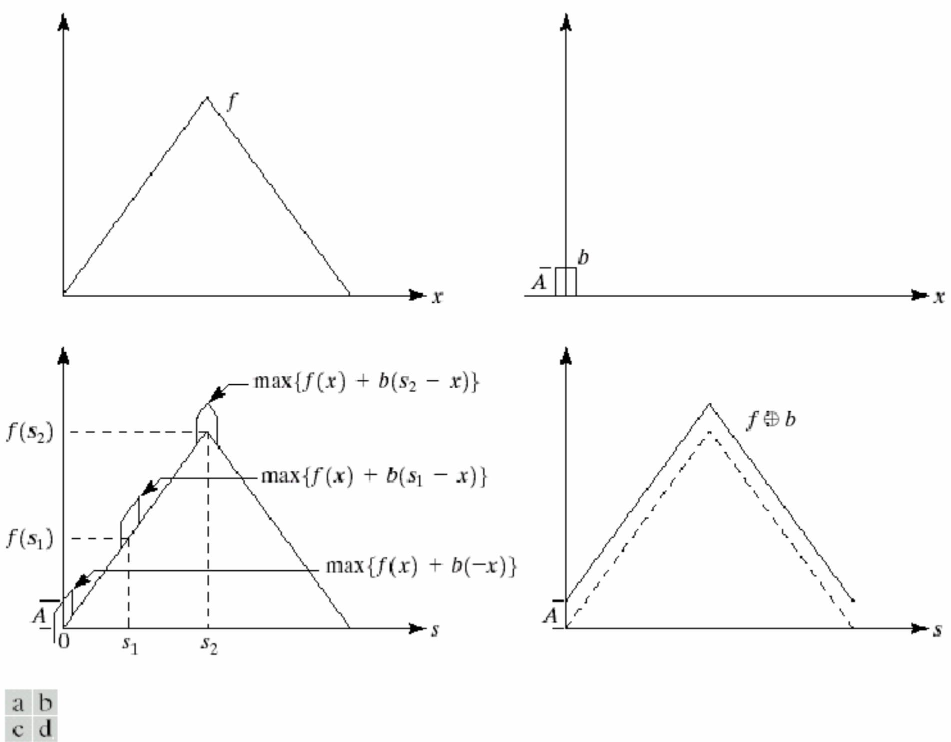


FIGURE 9.27 (a) A simple function. (b) Structuring element of height A. (c) Result of dilation for various positions of sliding b past f. (d) Complete result of dilation (shown solid).

General effect of dilation of a grey-scale image:

(1) If all values of $b(x,y)$ are positive $\rightsquigarrow$ output image brighter

(2) Dark details are reduced or eliminated, depending on size

9.6.2 Erosion

Grey-scale erosion of f by b, is defined as

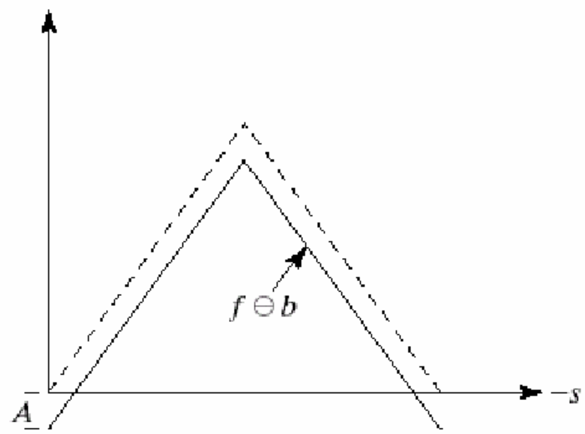
$$
\begin{aligned}
(f \ominus b)(s,t) = \max\{&f(s+x, t+y) - b(x,y) \\
&|(s+x),(t+y) \in D_f;\ (x,y) \in D_b\},
\end{aligned}
$$

where D_f and D_b are the domains of f and b, respectively

Simple 1D example. For functions of one variable:

$$
\begin{aligned}
(f \ominus b)(s) = \max\{&f(s+x) - b(x) \\
&|(s+x) \in D_f;\ x \in D_b\}
\end{aligned}
$$

FIGURE 9.28 Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).



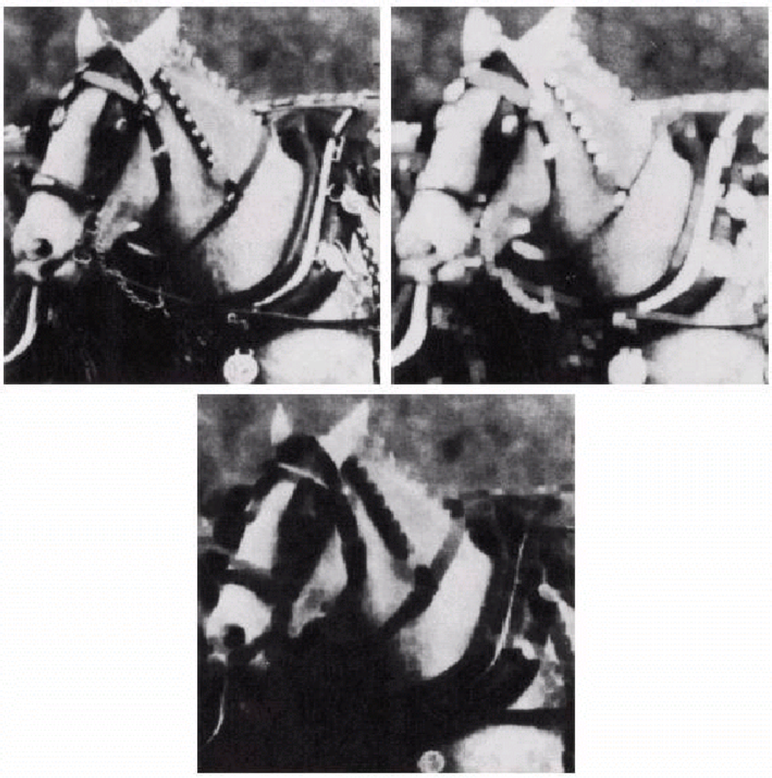
General effect of erosion of a grey-scale image:

(1) If all values of $b(x, y)$ are positive $\rightsquigarrow$ output image darker

(2) Bright details are reduced or eliminated, depending on size

Example 9.9: Dilation and erosion on grey-scale image

$f(x, y)$: 512 x 512
$b(x, y)$: "flat top", unit height, size of 5 x 5



a b
c

FIGURE 9.29 (a) Original image. (b) Result of dilation. (c) Result of erosion. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

9.6.3 Opening and closing

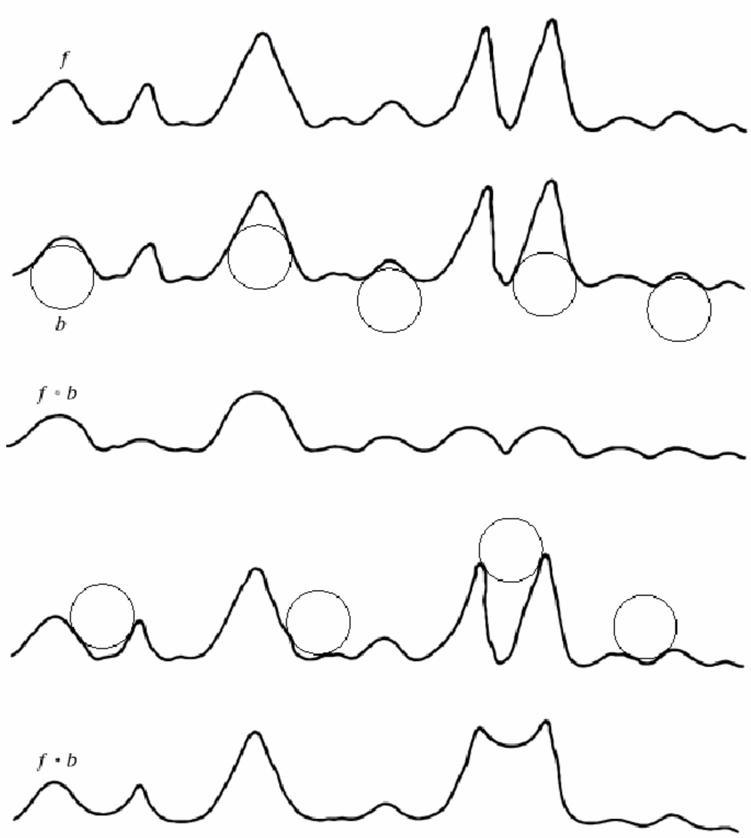
The **opening** of image f by b, is defined as

$$f \circ b = (f \ominus b) \oplus b$$

The **closing** of image f by b, is defined as

$$f \circ b = (f \oplus b) \ominus b$$

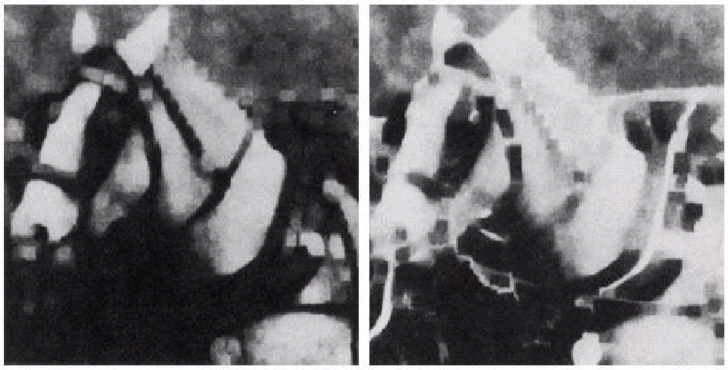
Explanation using "rolling ball":



a
b
c
d
e

FIGURE 9.30 (a) A gray-scale scan line. (b) Positions of rolling ball for opening. (c) Result of opening. (d) Positions of rolling ball for closing. (e) Result of closing.

Opening and closing of “horse image”:



a b

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

9.6.4 Some applications of grey-scale morphology

Morphological smoothing

Opening followed by closing $\rightsquigarrow$ remove or attenuate bright and dark artifacts or noise

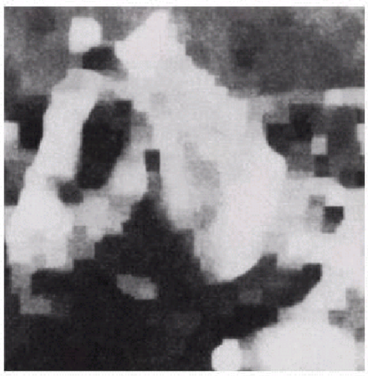


FIGURE 9.32 Morphological smoothing of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Morphological gradient

Definition: $g = (f \oplus b) - (f \ominus b)$

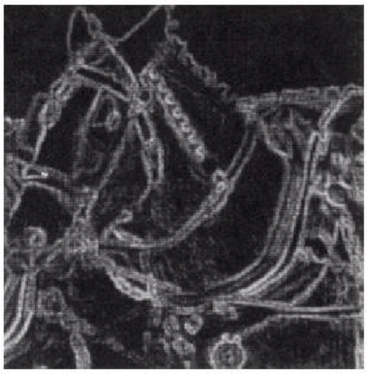


FIGURE 9.33 Morphological gradient of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Top-hat transformation

Definition: $h = f - (f \circ b)$

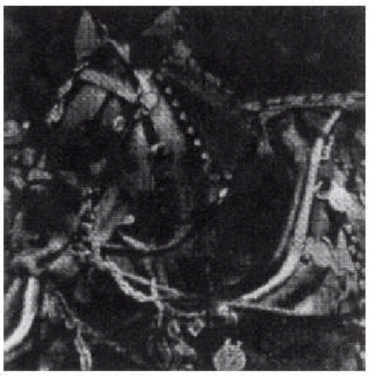
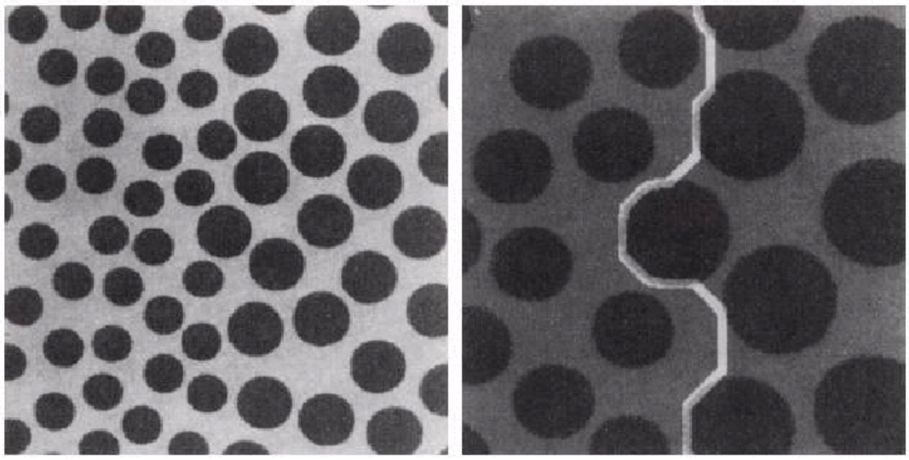


FIGURE 9.34 Result of performing a top-hat transformation on the image of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Textural segmentation

a b

FIGURE 9.35 (a) Original image. (b) Image showing boundary between regions of different texture. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)



Input image (left): Two texture regions

Output image (right): Boundary between the two regions

Algorithm:

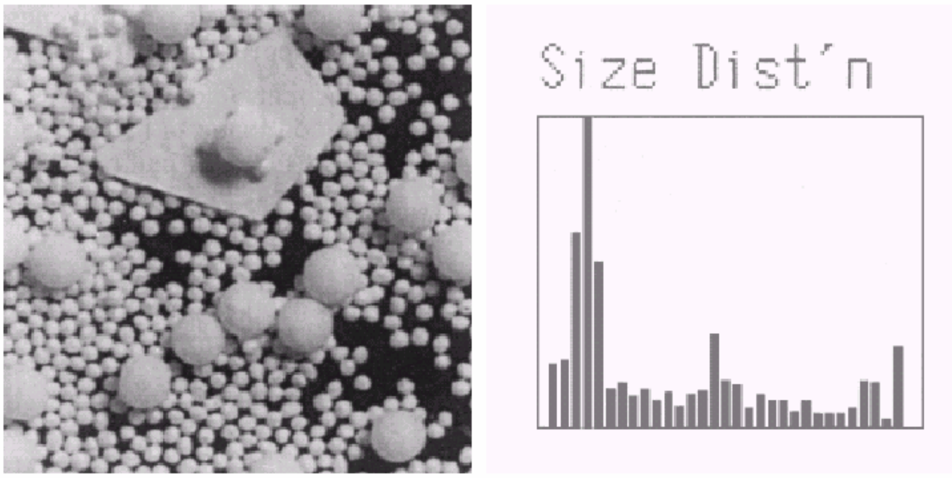
(1) Close input image using succ larger struct elements. When size(struct element) $\approx$ size(small blobs), blobs are removed

(2) Single opening with struct element that is large in relation to separation between large blobs $\rightsquigarrow$ light patches between blobs removed $\rightsquigarrow$ light region on left, dark region on right

(3) Thresholding $\rightsquigarrow$ boundary

Granulometry

Granulometry: Field that deals with determining the size distribution of particles in an image



a b

FIGURE 9.36 (a) Original image consisting of overlapping particles; (b) size distribution. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Algorithm:

(1) Opening with struct elements of increasing size

(2) Difference between original image and its opening computed after each pass

(3) Differences are normalized and used to construct histogram

The histogram indicates the presence of three predominant particle sizes