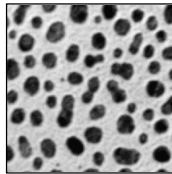# Morphology

Gray scale

161

Grayscale Morphology

```
g2 = ordfilt2(g1,n,se);
```

Order filter: replace the central pixel by the n-th element of the sorted image neighborhood pixel list.

M = sum(se(:) > 0);   % number of positive elements

n = 1:   minimum (erosion)

n = M: maximum (dilation)

n = M/2: median

se

$$\begin{matrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & ① & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{matrix}$$

31

Gray Scale Morphology

Replace dilation/erosion by maximum/minimum

dilation

$$(f \oplus b)(s,t) = \max\{f(s-x, t-y) + b(x,y)$$
$$|(s-x),(t-y) \in D_f, (x,y) \in D_b\}$$

$D_f = $ domain of f

$D_b = $ domain of b

f, b are functions not sets

One Dimension

$$(f \oplus b)(s) = \max\{f(s-x) + b(x)$$
$$|(s-x) \in D_f, x \in D_b\}$$

Note, we shift f rather than the structuring element.
Usually, D_b is smaller than D_f (figure 9:36)

smoothing is accomplished by opening followed by closing

(figure 9:38)

gradient: $g = (f \oplus b) - (f \ominus b)$ (figure 9:39)

top-hat: $h = f - (f \circ b)$ (figure 9:40)

Gray-value morphological processing

The techniques of morphological filtering can be extended to gray-level images. To simplify matters we will restrict our presentation to structuring elements, **B**, that comprise a finite number of pixels and are convex and bounded. Now, however, the structuring element has gray values associated with every coordinate position as does the image **A**.

* *Gray-level dilation*, D_G(*), is given by:

$$\text{Dilation -} \quad D_G(A,B) = \max_{[j,k] \in B} \{a[m-j,n-k] + b[j,k]\}$$

For a given output coordinate [m,n], the structuring element is summed with a shifted version of the image and the maximum encountered over all shifts within the $J \times K$ *domain* of **B** is used as the result. Should the shifting require values of the image **A** that are outside the $M \times N$ domain of **A**, then a decision must be made as to which model for image extension, as described in Section 9.3.2, should be used.

* *Gray-level erosion*, E_G(*), is given by:

$$\text{Erosion -} \quad E_G(A,B) = \min_{[j,k] \in B} \{a[m+j,n+k] - b[j,k]\}$$

The duality between *gray-level erosion* and *gray-level dilation*--the gray-level counterpart of eq. --is somewhat more complex than in the binary case:

$$\text{Duality -} \quad E_G(A,B) = -D_G(-\tilde{A},B)$$

where "$-\tilde{A}$" means that $a[j,k] \rightarrow -a[-j,-k]$.

The definitions of higher order operations such as *gray-level opening* and *gray-level closing* are:

$$\text{Opening -} \quad O_G(A,B) = D_G(E_G(A,B),B)$$
$$\text{Closing -} \quad C_G(A,B) = -O_G(-A,-B)$$

The important properties that were discussed earlier such as idempotence, translation invariance, increasing in A, and so forth are also applicable to gray level morphological processing. The details can be found in Giardina and Dougherty .

In many situations the seeming complexity of gray level morphological processing is significantly reduced through the use of symmetric structuring elements where $b[j,k] = b[-j,-k]$. The most common of these is based on the use of **B** = *constant* = 0. For this important case and using again the domain [j,k] $\subset$**B**, the definitions above reduce to:

$$\text{Dilation -} \quad D_G(A,B) = \max_{[j,k]\in B}\{a[m-j,n-k]\} = \max_B(A)$$

$$\text{Erosion -} \quad E_G(A,B) = \min_{[j,k]\in B}\{a[m-j,n-k]\} = \min_B(A)$$

$$\text{Opening -} \quad O_G(A,B) = \max_B(\min_B(A))$$

$$\text{Closing -} \quad C_G(A,B) = \min_B(\max_B(A))$$

The remarkable conclusion is that the *maximum filter* and the *minimum filter*, introduced in Section 9.4.2, are gray-level dilation and gray-level erosion for the specific structuring element given by the shape of the filter window with the gray value "0" *inside* the window. Examples of these operations on a simple one-dimensional signal are shown in Figure 45.
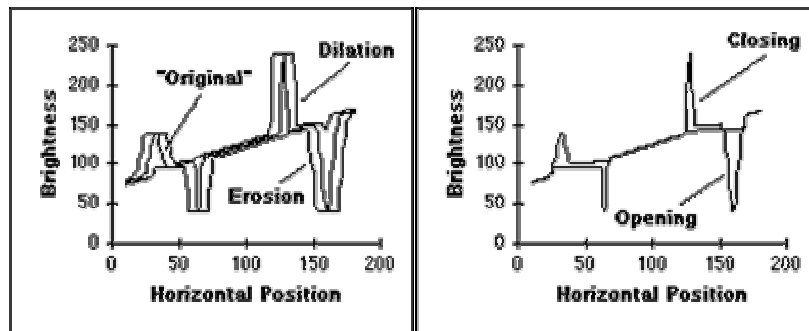
a) Effect of 15 x 1 *dilation* and *erosion* **b)** Effect of 15 x 1 *opening* and *closing*

Figure 45: Morphological filtering of gray-level data.

For a rectangular window, *J* x *K*, the two-dimensional maximum or minimum filter is separable into two, one-dimensional windows. Further, a one-dimensional maximum or minimum filter can be written in incremental form. (See Section 9.3.2.) This means that gray-level dilations and erosions have a computational complexity per pixel that is *O*(*constant*), that is, independent of *J* and *K*. (See also Table 13.)

The operations defined above can be used to produce morphological algorithms for smoothing, gradient determination and a version of the Laplacian. All are constructed from the primitives for *gray-level dilation* and *gray-level erosion* and in all cases the *maximum* and *minimum* filters are taken over the domain $[j,k]\in B$.

Morphological smoothing

This algorithm is based on the observation that a *gray-level opening* smoothes a gray-value image from above the brightness surface given by the function *a*[*m,n*] and the *gray-level closing* smoothes from below. We use a structuring element **B** based on eqs. and .

$$MorphSmooth(A,B) = C_G(O_G(A,B),B)$$
$$= \min(\max(\max(\min(A))))$$

Note that we have suppressed the notation for the structuring element **B** under the *max* and *min* operations to keep the notation simple.

Morphological gradient

For linear filters the gradient filter yields a vector representation (eq. (103)) with a magnitude (eq. (104)) and direction (eq. (105)). The version presented here generates a morphological estimate of the *gradient magnitude*:

$$Gradient(A,B) = \frac{1}{2}\left(D_G(A,B) - E_G(A,B)\right)$$

$$= \frac{1}{2}\left(\max(A) - \min(A)\right)$$

Morphological Laplacian

The morphologically-based Laplacian filter is defined by:

$$Laplacian(A,B) = \frac{1}{2}\left(\left(D_G(A,B) - A\right) - \left(A - E_G(A,B)\right)\right)$$

$$= \frac{1}{2}\left(D_G(A,B) + E_G(A,B) - 2A\right)$$

$$= \frac{1}{2}\left(\max(A) + \min(A) - 2A\right)$$

Summary of morphological filters

The effect of these filters is illustrated in Figure 46. All images were processed with a 3 x 3 structuring element as described in eqs. through . Figure 46e was contrast stretched for display purposes using eq. (78) and the parameters 1% and 99%. Figures 46c,d,e should be compared to Figures 30, 32, and 33.

a) Dilation **b)** Erosion **c)** Smoothing
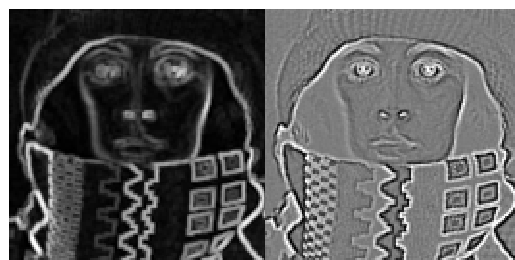
d) Gradient **e)** Laplacian

Figure 46: Examples of gray-level morphological filters.

The use of multiple grey levels introduces an enormous complication, both conceptually and computationally. A pixel can now have any integer value, so the nice picture of an image being a set disappears. There is also some question about what dilation, for example, should *mean* for a grey-level image. Rather than being strictly mathematical here we will take a more intuitive approach, in the hope that the result will make sense.

Consider the image of a line in Figure 2.22a. This is a bilevel image, and the dilation of this image by simple can be computed (Figure 2.22b). Now imagine that instead of having levels 0 and 1, the pixels in the line have the value 20 and the background is 0. What should a dilation of this new image by simple look like? The binary dilation spread out the line, as determined by the locations of the ''1'' pixels, making it three pixels wide instead of only one. The grey-level image should have a corresponding appearance after dilation, where the difference between the foreground and background pixels should be about the same as in the original and the line should be about three pixels wide. An example of how the dilated grey-level line (Figure 2.22c) might appear is given in Figure 2.22d.

This appears to be a reasonable analogue of dilation for the grey-level case, at least for a simple image. The image in Figure 2.22d was computed from Figure 2.22c in the following way:

$$(A \oplus S)[i,j] = \max\{A[i - r, j - c] +$$
$$S[r,c][i - r, j - c] \in A, [r,c] \in S\} \quad \text{(EQ 2.22)}$$

where S is the simple structuring element and A is the grey-level image to be dilated. This is one definition of a grey-scale dilation, and it can be computed as follows:
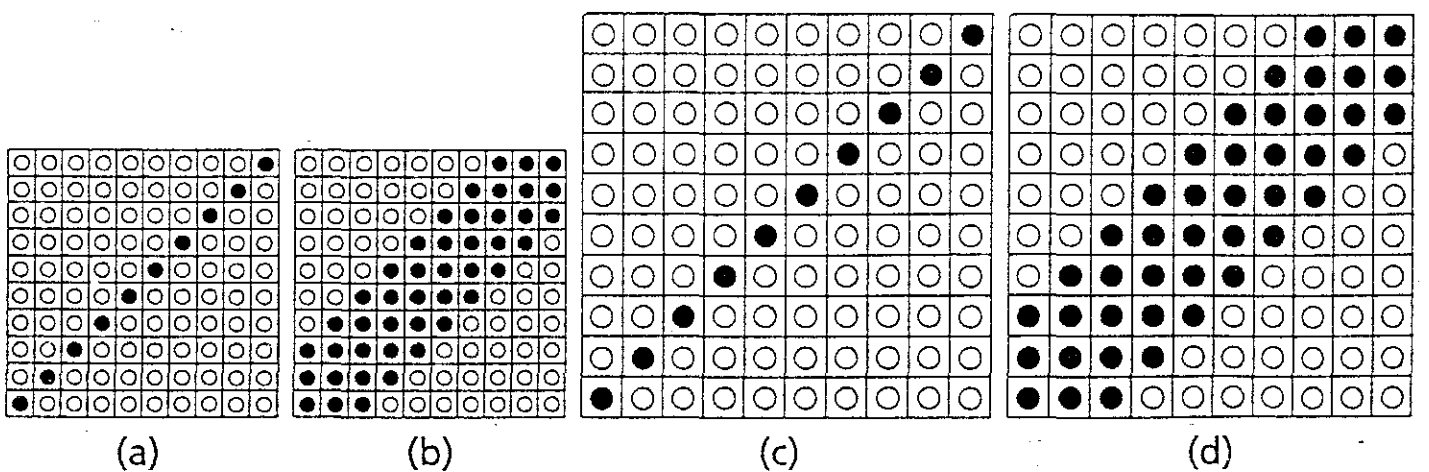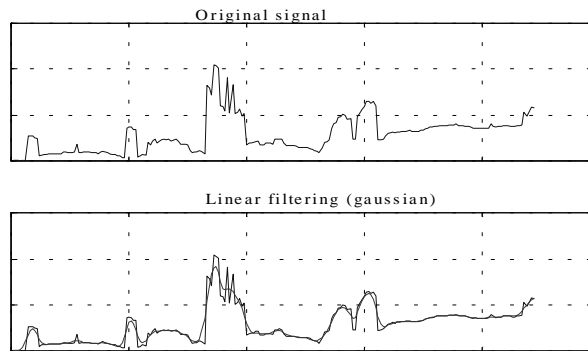

Figure 2.22 Grey-scale dilation. (a) A bilevel image of a line. (b) Binary dilation of (a) by simple. (c) A grey-scale image of a line; background is 0, line pixels have the value 20. (d) This is what the grey line should look like after a dilation.
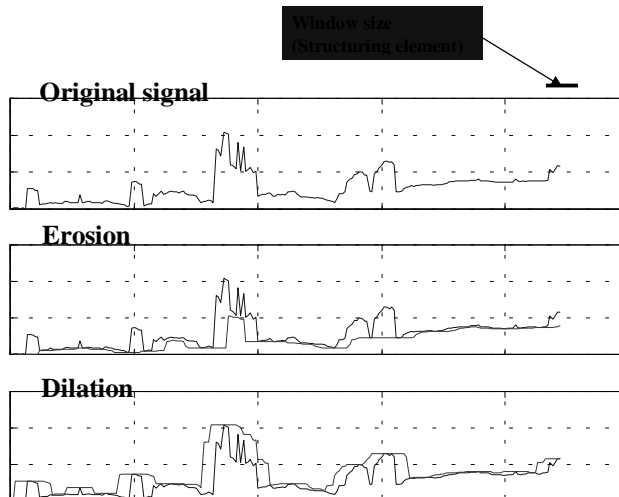
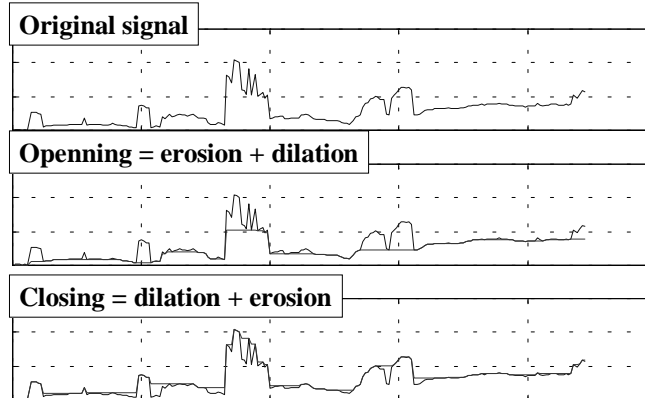# Grayscale Morphology – 1D example

Original signal

Linear filtering (gaussian)

167

Grayscale Morphology – 1D example (2)

Window size
(Structuring element)

Original signal

Erosion

Dilation

Grayscale Morphology – 1D example (3)

Original signal

Openning = erosion + dilation

Closing = dilation + erosion

169

Grayscale Morphology

Erosion

Dilation

Black   255
White   0
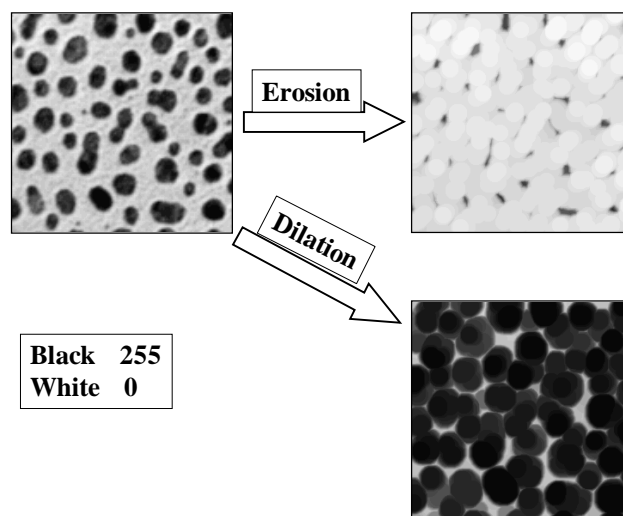
170

Strel shapes

se = strel(shape,parameters);

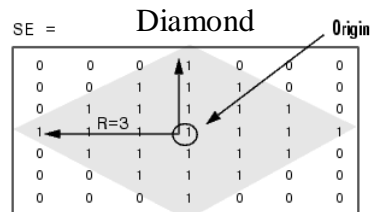| Flat Structuring Elements	Nonflat Structuring Elements
'arbitrary'	'arbitrary'
'pair'	'ball'
'diamond'	
'periodicline'	
'disk'	
'rectangle'	
'line'	
'square'	
'octagon'	

165

Strel shapes

Disk

Diamond

Line

Octagon

166

33

Grey scale morphology
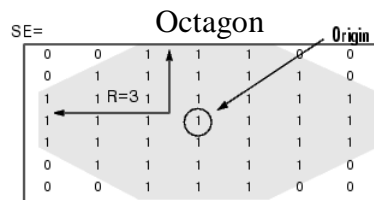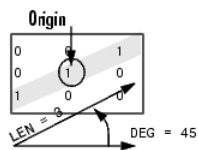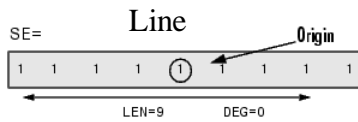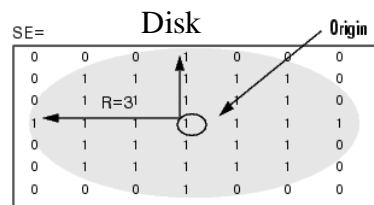
Dilation and erosion

Extending the concepts of binary image morphology to grey scale imagery i non-trivial. The images can no longer be regarded as point sets and the questio that immediately arises is: what do such operations as dilation mean wit respect to a grey scale image? A definition of grey scale dilation can be forme by extending the binary definition by use of a maximum operator:

$$(I \oplus X)(i,j) = \max\{I(i - x, j - y) + X(x,y) \mid (i - x),(j - y) \in D_I; (x,y) \in D_X\}$$

where D_I and D_X are the domains of I and X and both I and X are now functior of two spatial variables.

The algorithmic definition of this operation is perhaps more accessible and defined as:

for each pixel:

position the structuring element over the pixel and calculate the sum each pair of corresponding pixels

find the maximum of the sums and output this value

Note the similarity of this operation to cross-correlation. Here, instead of calc lating the sum of products over the corresponding mask and image elements v

Figure 13.10
A one-dimensional demonstration of grey-scale erosion (adapted from an illustration by Gonzalez and Woods (1992)).

$I(i)$ $X(x)$ $I \ominus X$
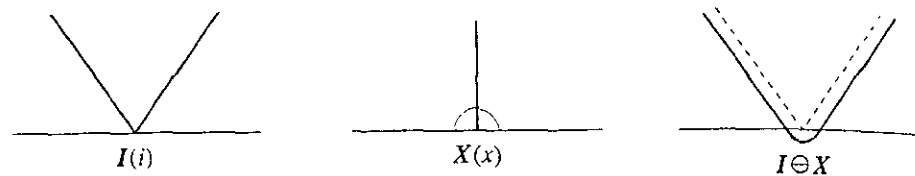
select the maximum of the sums of corresponding pixels in the image and str turing element. We can describe the effect of the operation qualitatively in th following terms. First, because of the addition operation and the selection of th maximum sum, the image being dilated will tend to be brighter (assuming po itive values in the structuring element). Second, dark details will tend to t reduced and relatively large dark structure will be undisturbed. This eff depends on the nature and extent of the structuring element with respect image structure or detail. A simple one-dimensional illustration is shown Figure 13.9. In this case there is no shape information in the structuring el ment, but the effect of the grey scale variation in the structuring element on t image is clear.

Grey scale erosion can be similarly defined. In this case we extend the bin definition by using a minimum operator and subtraction:

$$(I \ominus X)(i,j) = \min\{I(i + x, j + y) - X(x,y) \mid (i + x),(j + y) \in D_I; (x,y) \in D_X\}$$

where D_I and D_X are the domains of I and X and both I and X are now functi of two spatial variables. Figure 13.10 is a simple one-dimensional effect of t operation. The effect on an image is first, to tend to make it darker becaus the subtraction, and bright details in the input image that are contained wit the structuring element are diminished.

simply replacing the set X with the subgraph of the grey tone image f:
$\varepsilon_B[\mathrm{SG}(f)] = \{(\mathbf{x}, t) \mid B_{(\mathbf{x}, t)} \subseteq \mathrm{SG}(f)\}$. Examples of erosions on binary and
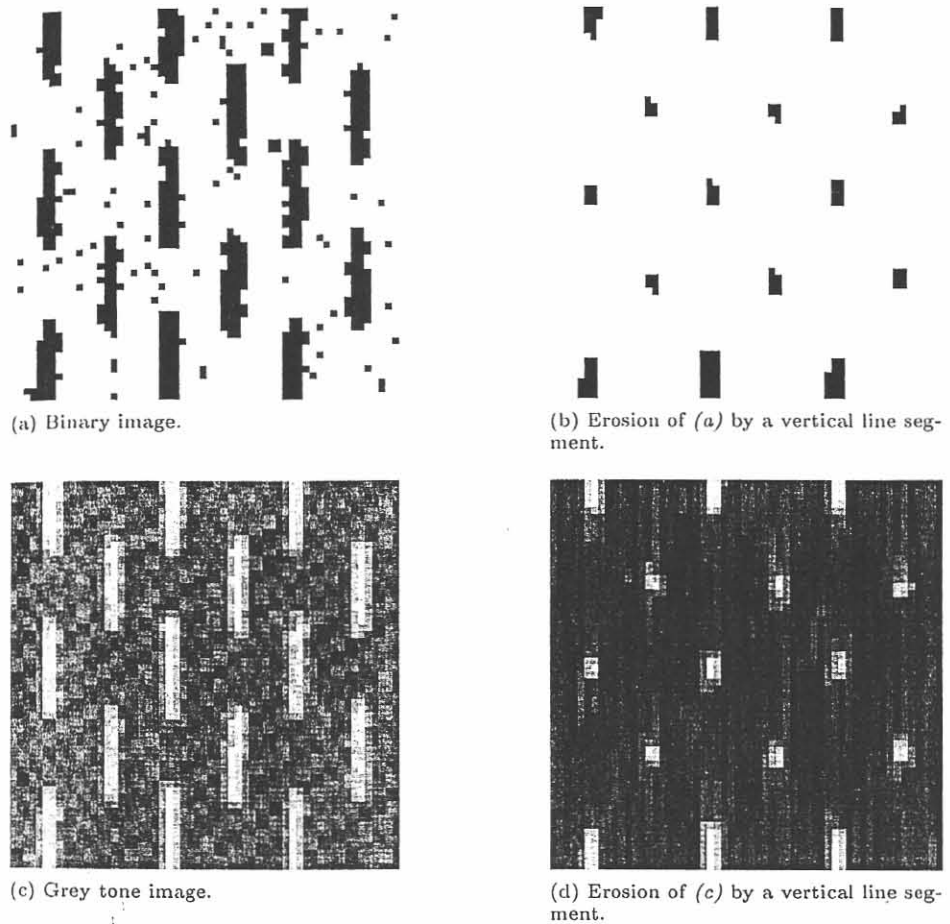grey tone images are shown in Fig. 3.5. In this figure, the eroded vertical bars

(a) Binary image.

(b) Erosion of (a) by a vertical line segment.

(c) Grey tone image.

(d) Erosion of (c) by a vertical line segment.

Fig. 3.5. Erosion of binary and grey tone images by a centred vertical line segment of 13 pixels.

are reduced to a few points because the structuring element could fit the bars only when it was centred on these points. The bars connected to the top and bottom image borders are not as much eroded as those lying within the image because it has been assumed that values outside the image definition domain are set to $t_{\max}$. This is a common way to deal with border effects when eroding an image (see local knowledge property of erosions, page 70).

Nonflat SEs have grey scale values for each point of their domain of definition. The erosion by a nonflat SE B_v is defined as follows:

68   3. Erosion and Dilation

$$\left[\varepsilon_{B_v}(f)\right](\mathbf{x}) = \min_{\mathbf{b} \in \mathcal{D}_{B_v}} \left\{ f(\mathbf{x} + \mathbf{b}) - B_v(\mathbf{b}) \right\}.$$

The grey scale weights of a volumic SE should be set according to the image intensity values. Because of the difficulty to meet this condition and due to the extra computations required by the subtraction, grey scale SEs are seldom used in practical applications.

3.3 Dilation

The dilation is the dual operator of the erosion and is based on the following question: *"Does the structuring element hit the set?"* The dilated set is the locus of points where the answer to this question is affirmative.

The dilation of a set X by a structuring element B is denoted by $\delta_B(X)$ and is defined as the locus of points $\mathbf{x}$ such that B hits X when its origin coincides with $\mathbf{x}$:

$$\delta_B(X) = \{\mathbf{x} \mid B_\mathbf{x} \cap X \neq \emptyset\}. \tag{3.4}$$

This definition is illustrated in Fig. 3.6 for the dilation of a set by a disc SE. Note that the dilation of a single pixel $\mathbf{x}$ outputs the reflected SE, centred
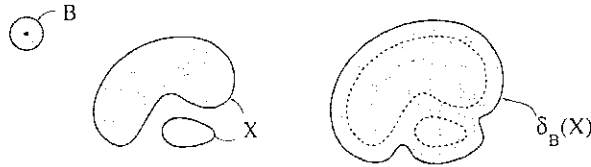
Fig. 3.6. Dilation δ of a set X by a disc B. The two connected components of X are connected by the dilation: B always hits X when it is placed in the channel separating the particles.

on $\mathbf{x}$: $\delta_B(\mathbf{x}) = \check{B}_\mathbf{x}$. Equation 3.4 can be rewritten in terms of a union of set translations, the translations being defined by the SE:

$$\delta_B(X) = \bigcup_{\mathbf{b} \in B} X_{-\mathbf{b}}.$$

This latter definition can be directly extended to binary and grey scale images: the dilation of an image f by a structuring element B is denoted by $\delta_B(f)$ and is defined as the maximum of the translation of f by the vectors $-\mathbf{b}$ of B:

$$\delta_B(f) = \bigvee_{\mathbf{b} \in B} f_{-\mathbf{b}}. \tag{3.5}$$

In other words, the dilated value at a given pixel x is the maximum value of the image in the window defined by the structuring element when its origin is at $\mathbf{x}$:

$$\left[\delta_B(f)\right](\mathbf{x}) = \max_{\mathbf{b} \in B} f(\mathbf{x} + \mathbf{b}). \tag{3.6}$$

The dilation of a 1-D signal is illustrated in Fig. 3.7. Examples of dilations

(a) 1-D signal f and SE B.

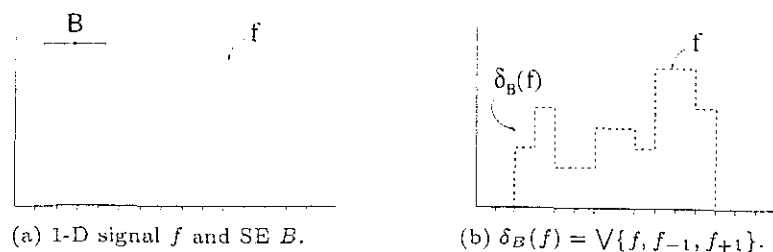(b) $\delta_B(f) = \bigvee\{f, f_{-1}, f_{+1}\}$.

Fig. 3.7. Dilation represented as the point-wise maximum of a series of image translations (the successive image translations are shown in Fig. 3.4). The dilated value of a given pixel corresponds to the maximum value of the image within the neighbourhood defined by the SE when it is centred at this pixel. Note that the following equation also holds: $\delta_B[SG(f)] = \{(\mathbf{x}, t) \mid B_{...} \cap SG(f) \neq \emptyset\}$.

on binary and grey tone images are shown in Fig. 3.8. Note that the SE is long enough to hit the line segments of the dashed line when it is located in between two segments. This explains why the gaps are completely filled in the dilated images.
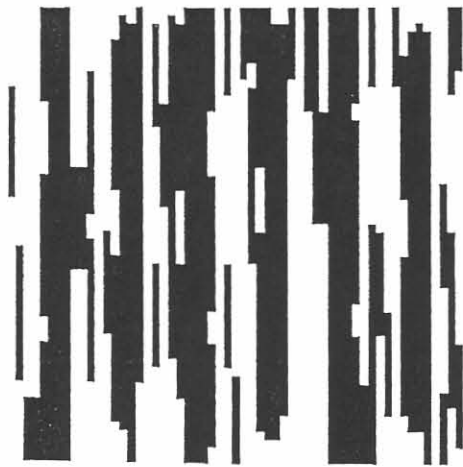
(a) Dilation of Fig. 3.5a by a vertical line segment.
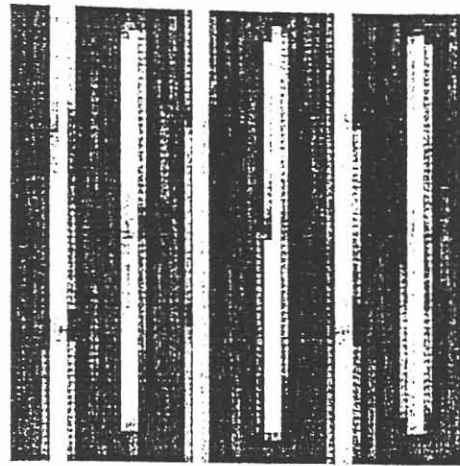
(b) Dilation of Fig. 3.5c by a vertical line segment.

Fig. 3.8. Dilation of binary and grey tone images by a centred vertical line segment of 13 pixels (the input images are displayed in Figs. 3.5a and c respectively). The displayed dilations have been computed assuming that values outside the image definition domain are set to 0.

We denote by nB a structuring element of size n, i.e., a SE B that has been dilated ~ ... a by the ... l SE $\check{B}$ (... =

$$nB = \delta_{\check{B}}^{(n-1)}(B), \tag{3.7}$$

where $n \geq 2$. When B is convex, then nB is the stretching of B with size parameter n. An elementary triangle and the corresponding triangles of size 2 and 3 are shown in Fig. 3.9.
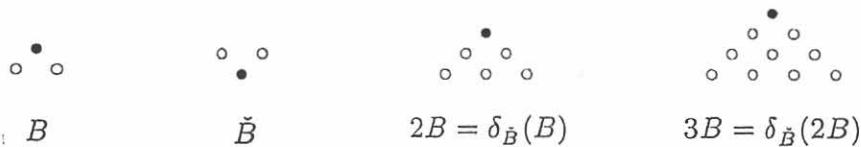

$$B \qquad\qquad \check{B} \qquad\qquad 2B = \delta_{\check{B}}(B) \qquad\qquad 3B = \delta_{\check{B}}(2B)$$

Fig. 3.9. An elementary triangle B, its reflection $\check{B}$, and the corresponding structuring elements of size 2 and 3 obtained by dilating B once and twice with $\check{B}$.

The dilation by a nonflat SE B_v is defined as follows:

$$\left[\delta_{B_v}(f)\right](x) = \max_{b \in \mathcal{D}_{B_v}} \left\{ f(x + b) + B_v(b) \right\}.$$

Unlike images dilated or eroded by flat SEs, grey scale values of images dilated or eroded by nonflat SEs are not bounded by the minimum and maximum grey scale values of the input image. In particular, erosions with nonflat SEs may output negative integers.

4.5.1 Definitions

The *white top-hat*[3] WTH of an image f is the difference between the original image f and its opening γ:

$$\text{WTH}(f) = f - \gamma(f), \qquad (4.9)$$

i.e., WTH = id $- \gamma$. Since the opening is an anti-extensive image transformation, the grey scale values of the white top-hat are always greater or equal to zero. An example is shown in Fig. 4.15 for a 1-D signal. Note that the signal peaks are extracted independently from their intensity level. It is only a

---
[3] White top-hats are also called *top-hats by opening*.

(a) Original image f and its opening γ by B.

(b) $\text{WTH}(f) = f - \gamma_B(f)$.
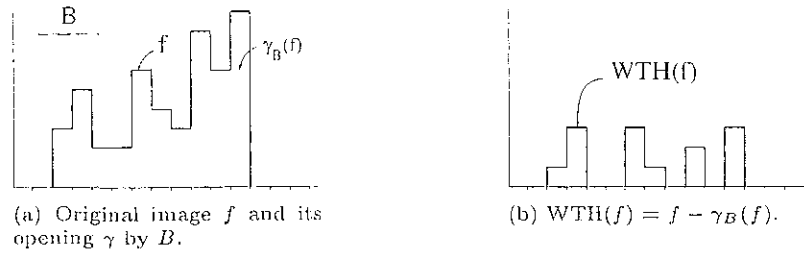
Fig. 4.15. White top-hat WTH of a 1-D signal.

shape criterion that is taken into account: all structures of the subgraph that cannot contain the SE are extracted by the white top-hat. White top-hats are non-increasing but idempotent transformations:

$$\text{WTH}[\text{WTH}(f)] = f - \gamma(f) - \underbrace{\gamma[f - \gamma(f)]}_{\emptyset}$$
$$= f - \gamma(f).$$

The dual transformation of the white top-hat with respect to set complementation is:

$$[\text{WTH}(f^c)]^c = t_{\max} - \text{WTH}(t_{\max} - f)$$
$$= t_{\max} - t_{\max} + f + \underbrace{\gamma(t_{\max} - f)}_{[\phi(f)]^c}$$
$$= t_{\max} + f - \phi(f).$$

In practice however, the *black top-hat*[4] BTH of an image f is simply defined as the difference between the closing ϕ of the original image and the original image:

$$\text{BTH}(f) = \phi(f) - f, \qquad (4.10)$$

i.e., BTH = $\phi -$ id. It follows that black and white top-hats are complementary operators: BTH = WTHC. Owing to the extensivity property of the closing operator, the values of the black top-hat images are always greater or equal to zero. It is neither an idempotent nor an increasing transformation. A black top-hat is illustrated in Fig. 4.16 for a 1-D signal.

The sum of the white and black top-hats extracts all image structures that cannot contain the SE *whatever* their relative contrast (i.e., peaks and troughs). In fact, this sum comes down to the arithmetic difference between the closing and the opening of the image. Owing to its self-complementariness property, we call this operator the *self-complementary top-hat* and denote it by ϱ:

$$\varrho = \text{WTH} + \text{BTH} = \phi - \gamma.$$

---
[4] Black top-hats are also called *top-hats by closing*.

(a) Original image f and its closing o by B.

$$BTH(f) = \phi_B(f) - f$$

(b) $BTH(f) = o_B(f) - f$.

Fig. 4.16. Black top-hat BTH performed on a 1-D signal. Note that the troughs are extracted independently of their intensity levels.

The shape and size of the structuring element used for top-hat transforms depend on the morphology of the structures to be extracted. For instance, if we want to detect bright features of width smaller than l, a white top-hat with a disc structuring of size slightly larger than l should be considered so as to remove them in the opening or closing step (opening for bright and closing for dark objects with respect to their background). For example, Fig. 4.17 illustrates the extraction of one pixel thick bright structures using a top-hat by opening. These structures are first removed from the input image by a

(a) Input image f

(b) Parametric opening $\gamma_{\blacksquare_{3},6}(f)$

(c) Top-hat by opening $f - \gamma_{\blacksquare_{3},6}(f)$
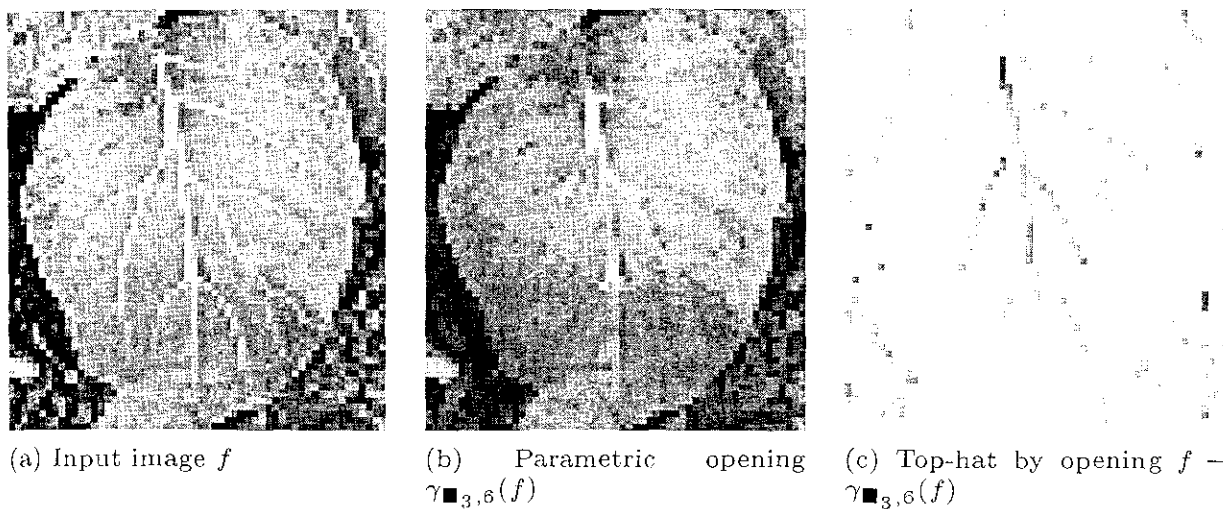
Fig. 4.17. Extraction of bright one pixel thick structures using a top-hat by opening.

parametric opening with a 3×3 square while allowing for gaps of up to 3 pixels (i.e., $\lambda = 6$ in Eq. 4.5). The structures themselves are then recovered by the corresponding top-hat by opening. In this example, a parametric rather than a plain opening produces better results because it is more selective for removing the targeted structures.

In situations where the input image is corrupted by a high frequency noise signal, it must be filtered out *before* using top-hat transforms to avoid side effects. For example, a closing by a small SE should be considered before computing a white top-hat and an opening before a black top-hat.

Functions - Gray

conndef	Default connectivity array
imbothat	Perform bottom-hat filtering
imclearborder	Suppress light structures connected to image border
imclose	Close image
imdilate	Dilate image
imerode	Erode image
imextendedmax	Extended-maxima transform
imextendedmin	Extended-minima transform
imfill	Fill image regions
imhmax	H-maxima transform
imhmin	H-minima transform
imimposemin	Impose minima
imopen	Open image
imreconstruct	Perform morphological reconstruction
imregionalmax	Regional maxima of image
imregionalmin	Regional minima of image
imtophat	Perform tophat filtering
watershed	Find image watershed regions

149

BW Morphology (1)

bw2 = dilate(bw1,se);   % se   general structuring element

bw2 = erode(bw1,se);

bw2 = bwmorph(bw1,opt,N)   % opt defines a 3x3 se

```
'bothat'    Subtract the input image from its closing
'bridge'    Bridge previously unconnected pixels
'clean'     Remove isolated pixels
'close'     Perform binary closure
'diag'      Diagonal fill to eliminate 8-connectivity
'dilate'    Perform dilation
'erode'     Perform erosion
'fill'      Fill isolated interior pixels
'hbreak'    Remove H-connected pixels
'majority'
'open'      Perform binary opening
'remove'    findonly boundary pixels
'shrink'
'skel'      skeletonization
'spur'      Remove end points of lines
'thicken'
'thin'
'tophat'    Subtract the opening from the input image
```

150

Grayscale Morphology main functions

imbothat	Perform bottom-hat filtering
imclose	Close image
imdilate	Dilate image
imerode	Erode image
imfill	Fill image regions
imopen	Open image
imreconstruct	Perform morphological reconstruction
imtophat	Perform tophat filtering
watershed	Find image watershed regions
strel	structuring element

163
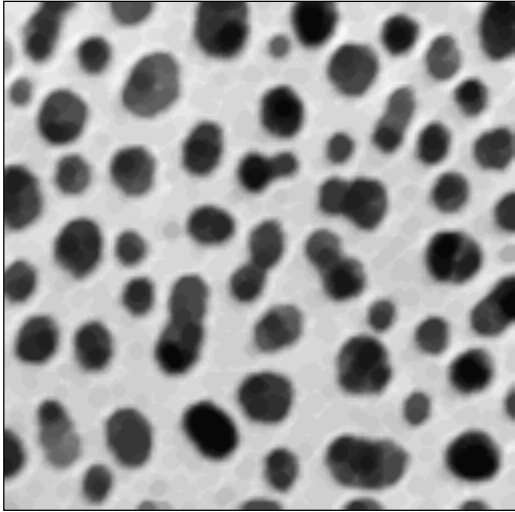
Strel: structure element

Creation and Manipulation

getheight	Get the height of a structuring element
getneighbors	Get structuring element neighbor locations and heights (coordinate list)
getnhood	Get structuring element neighborhood (matrix)
getsequence	Extract sequence of decomposed structuring elements
isflat	Return true for flat structuring element
reflect	Reflect structuring element
strel	Create morphological structuring element
Translate	Translate structuring element
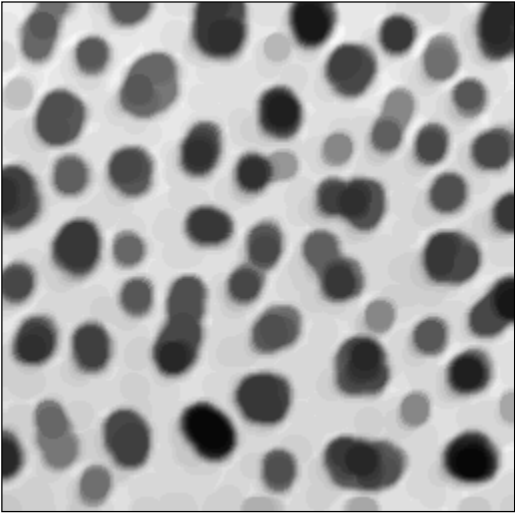
164

32

Grayscale Opening

Structuring
Element

171

Grayscale Opening

Structuring
Element

172

Grayscale Opening
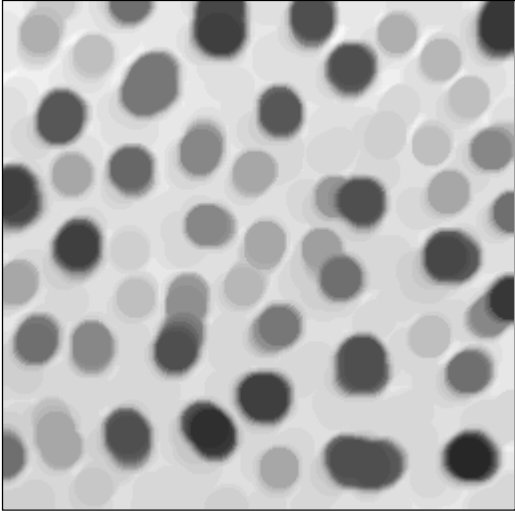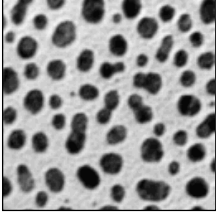
Structuring Element

173
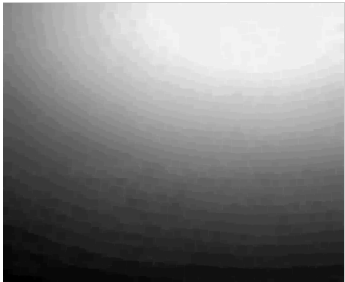

Grayscale example (1)

Step 1: background reconstruction via closing operation

```
x = ordfilt2(Letter,49,ones(7), symm ); % dilation
y = ordfilt2(x,1,ones(7),'symm');       % erosion
```

Letter   **y**
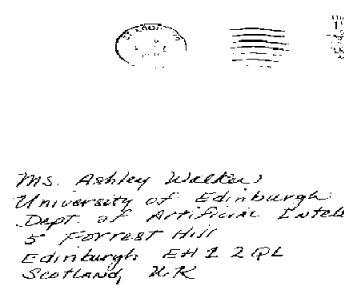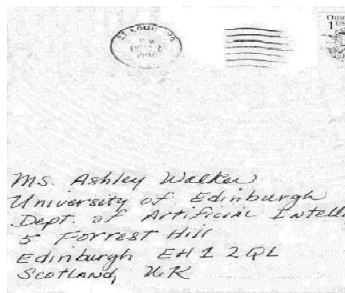

174

37

Grayscale example (2)

Step 2: background subtraction **Step 3: Thresholding**

Top hat (bottom hat) operation:

Z = Letter   y; **Z > Threshold**

175

Grayscale example (3)

Example taken from the demo files of the
SDC Morphology toolbox v0.13

176
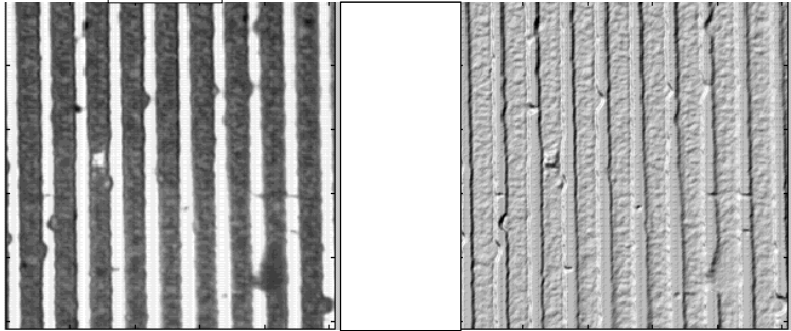
Defect Detection

The input image is a gray-scale image of a microelectronic circuit. The relevant objects in this image are vertical metal stripes. These stripes have some irregularities that should be detected.

Source

gradient projection
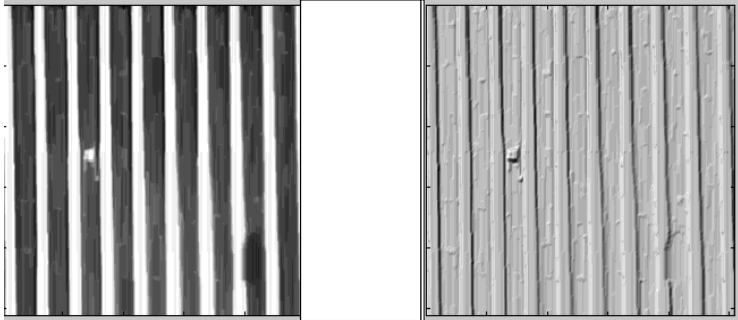
Closing

Closing of the image by a vertical line of length 25 pixels.

Close

surf

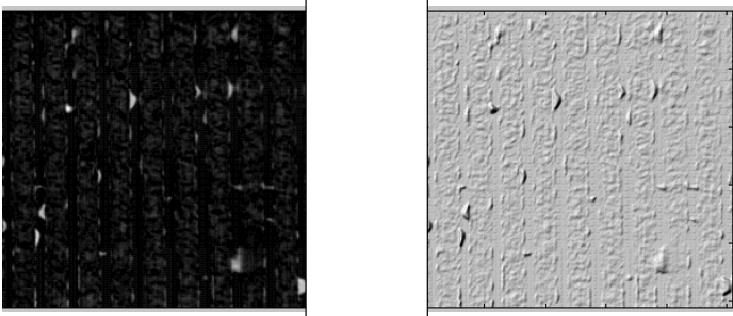# Closing Top-Hat

Subtraction of the closing from the original is called closing top-hat. It shows the discrepancies of the image where the structuring element cannot fit the surface. In this case, it highlights vertical depression with length longer than 25 pixels.
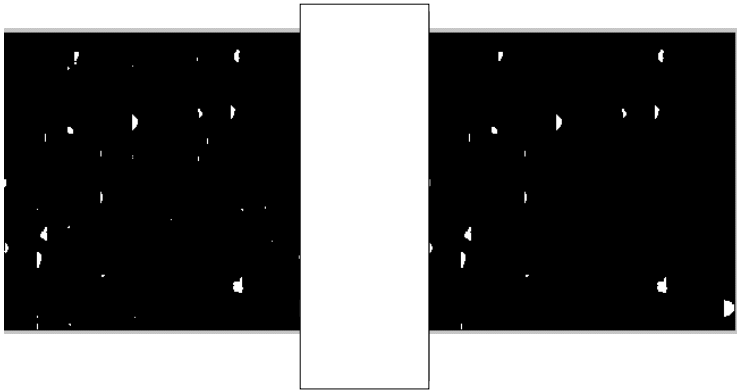
Close   **surf**
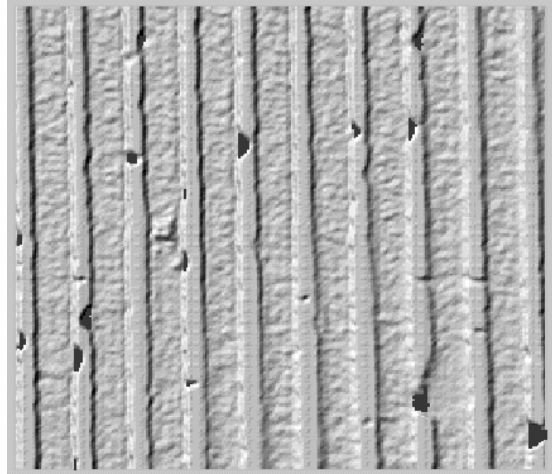

179

Thresholding & Cleaning

180

40

Result

181

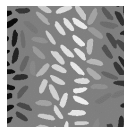
Segmentation

The idea of segmentation has its roots in work by Gestalt psychologist (e.g. Kohler) who studied the preferences exhibited by human beings in grouping or organizing sets of shapes arranged in the visual field.   (Ballard & Brown)

Segmentation is the first essential and important step of low-level vision   (Marr, Rosenfeld, Hall, Gonzalez, in Pal & Pal).

182

FIGURE 9.26 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

9.6 Extensions to grey-scale images

$$f(x, y): \text{ Input image}$$
$$b(x, y): \text{ Structuring element image}$$

9.6.1 Dilation

Grey-scale dilation of f by b, is defined as

$$(f \oplus b)(s, t) = \max \{ f(s - x, t - y) + b(x, y)$$
$$| (s - x), (t - y) \in D_f; (x, y) \in D_b \},$$

where D_f and D_b are the domains of f and b, respectively

Simple 1D example. For functions of one variable:

$$(f \oplus b)(s) = \max \{ f(s - x) + b(x)$$
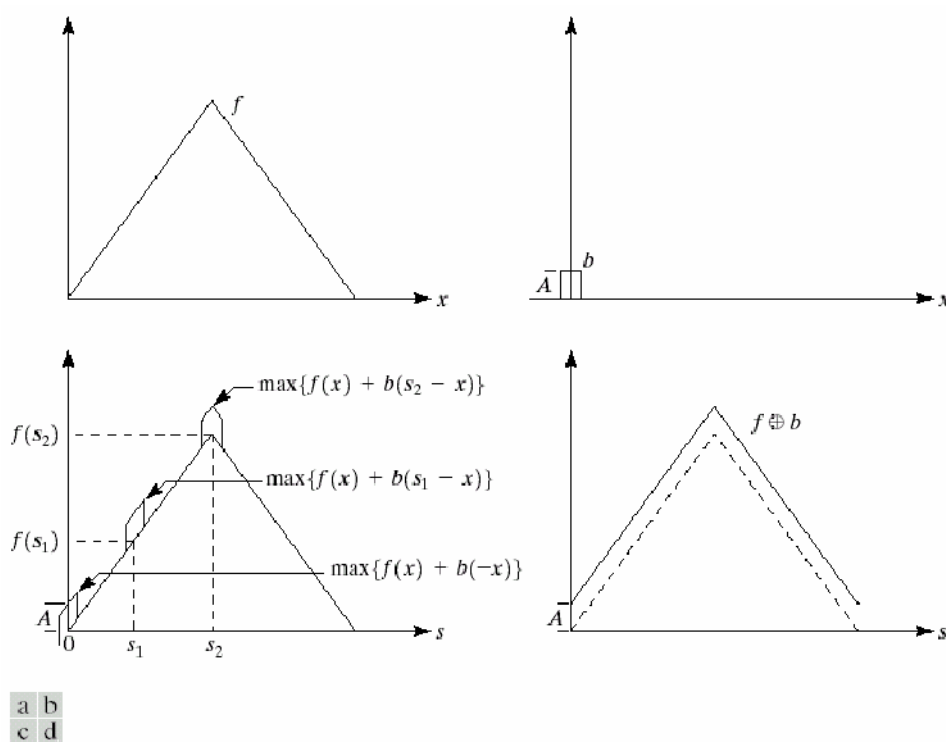$$| (s - x) \in D_f; \ x \in D_b\}$$

a b
c d

FIGURE 9.27 (a) A simple function. (b) Structuring element of height A. (c) Result of dilation for various positions of sliding b past f. (d) Complete result of dilation (shown solid).

General effect of dilation of a grey-scale image:

(1) If all values of $b(x, y)$ are positive $\rightsquigarrow$ output image brighter

(2) Dark details are reduced or eliminated, depending on size

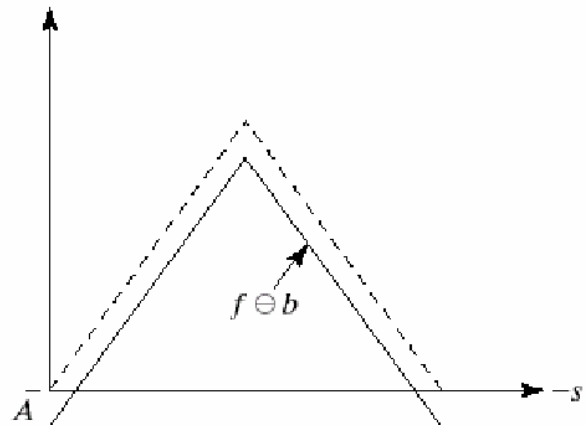## 9.6.2 Erosion

Grey-scale erosion of f by b, is defined as

$$(f \ominus b)(s,t) = \max \{ f(s+x, t+y) - b(x,y)$$
$$|(s+x), (t+y) \in D_f; (x,y) \in D_b\},$$

where D_f and D_b are the domains of f and b, respectively

Simple 1D example. For functions of one variable:

$$(f \ominus b)(s) = \max \{ f(s+x) - b(x)$$
$$|(s+x) \in D_f; x \in D_b\}$$

FIGURE 9.28
Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).

General effect of erosion of a grey-scale image:

(1) If all values of $b(x, y)$ are positive $\rightsquigarrow$ output image darker

(2) Bright details are reduced or eliminated, depending on size

Example 9.9: Dilation and erosion on grey-scale image

$$f(x, y):\ 512 \text{ x } 512$$
$$b(x, y):\ \text{"flat top", unit height, size of 5 x 5}$$

a b
c
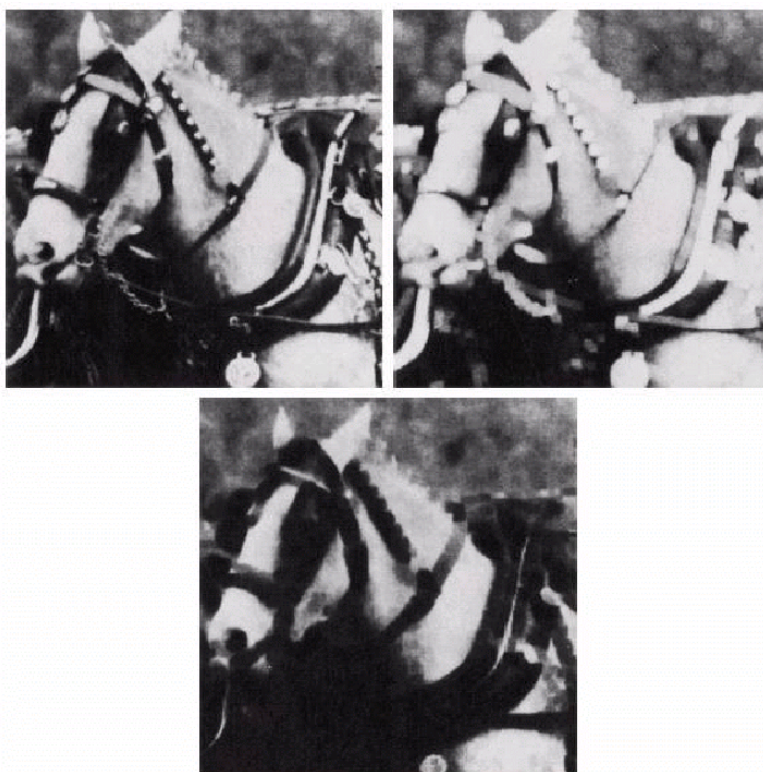
FIGURE 9.29
(a) Original
image. (b) Result
of dilation.
(c) Result of
erosion.
(Courtesy of
Mr. A. Morris,
Leica Cambridge,
Ltd.)

9.6.3 Opening and closing

The **opening** of image f by b, is defined as

$$f \circ b = (f \ominus b) \oplus b$$

The **closing** of image f by b, is defined as

$$f \circ b = (f \oplus b) \ominus b$$

Explanation using "rolling ball":
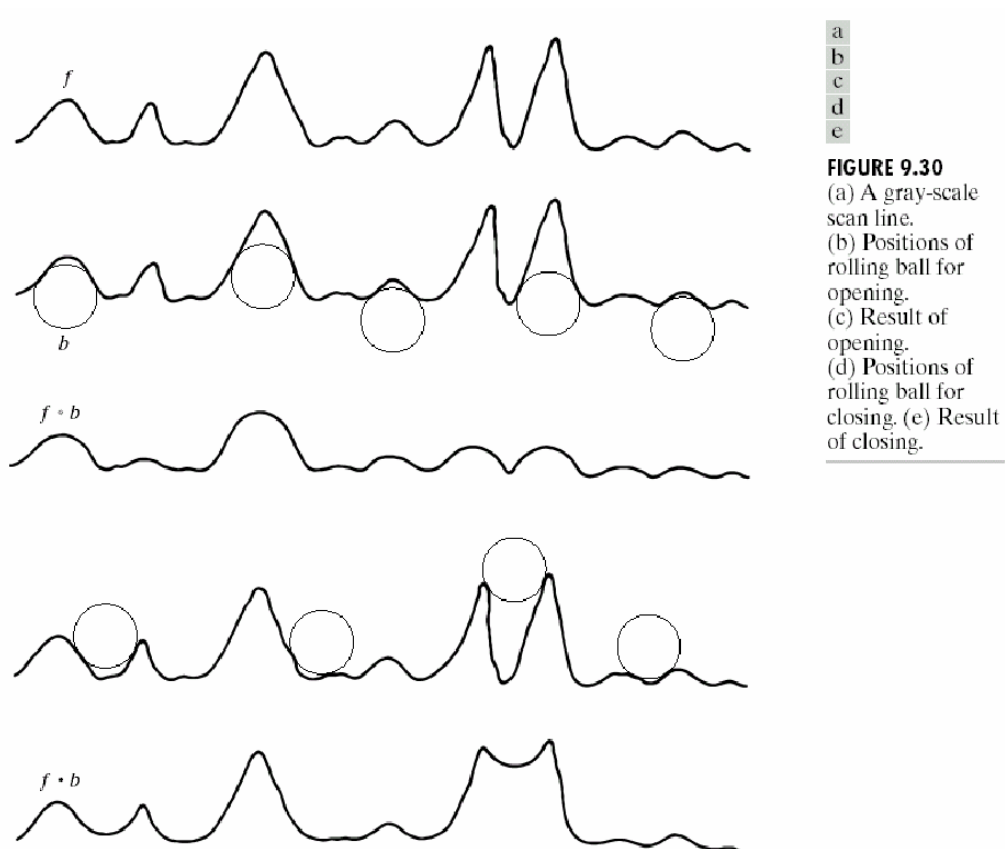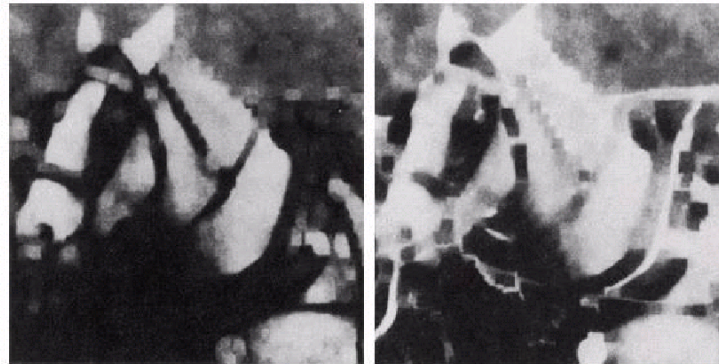

a
b
c
d
e

FIGURE 9.30
(a) A gray-scale scan line.
(b) Positions of rolling ball for opening.
(c) Result of opening.
(d) Positions of rolling ball for closing. (e) Result of closing.

Opening and closing of "horse image":

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

9.6.4 Some applications of grey-scale morphology

Morphological smoothing

Opening followed by closing $\rightsquigarrow$ remove or attenuate bright and dark artifacts or noise
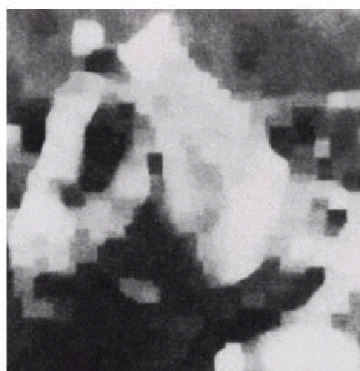

FIGURE 9.32 Morphological smoothing of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Morphological gradient

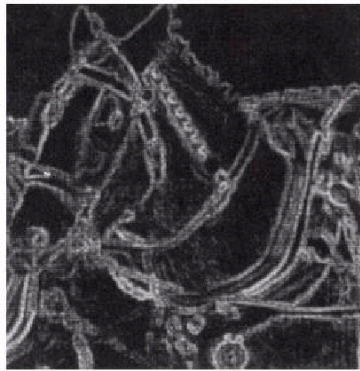Definition: $g = (f \oplus b) - (f \ominus b)$

FIGURE 9.33 Morphological gradient of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

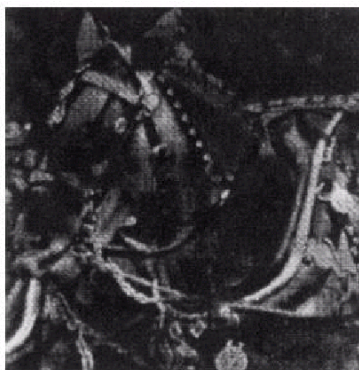## Top-hat transformation

Definition: $h = f - (f \circ b)$

FIGURE 9.34 Result of performing a top-hat transformation on the image of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

4.5.2 Application to the correction of uneven illumination

An illumination gradient occurs when a scene is unevenly illuminated. There is a need for correcting this effect because grey scale measurements and global threshold techniques cannot be applied to images of unevenly illuminated scenes. The best solution is to optimise the lighting system so as to acquire evenly illuminated images but still this is impossible in many practical situations. For instance, the background 'illumination' of an x-ray image of a manufactured metallic part of uneven width is directly proportional to the width of this part and is therefore uneven: the larger the width, the darker the output intensity level. If the image objects have all the same local contrast, i.e., if they are either all darker or brighter than the background, top-hat transforms can be used for mitigating illumination gradients. Indeed, a top-hat with a large isotropic structuring element acts as a high-pass filter. As the illumination gradient lies within the low frequencies of the image, it is removed by the top-hat. White top-hats are used for dark backgrounds and black top-hats for bright backgrounds. For example, the upper left image of Fig. 4.18 presents an increasing illumination from left to right. Global
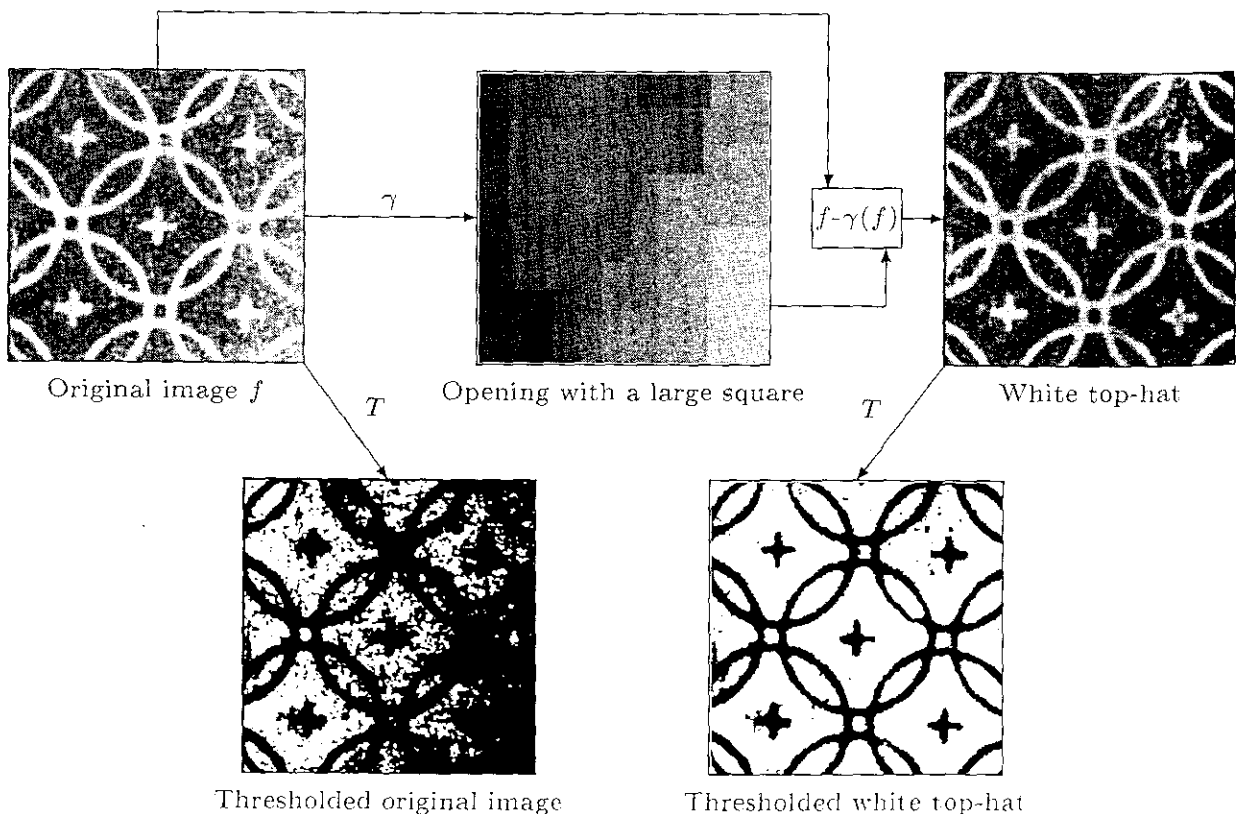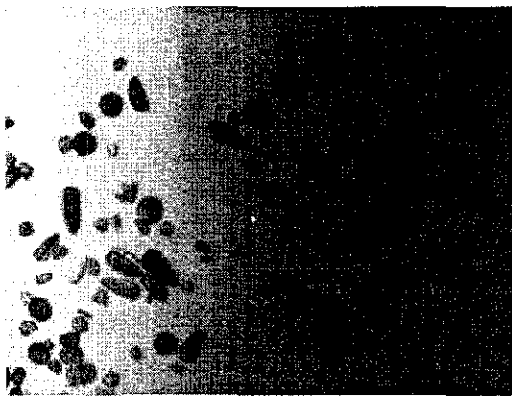

Fig. 4.18. Use of top-hat for mitigating inhomogeneous illumination. The performance of this technique is illustrated by the thresholds on the original and top-hat images.

thresholding techniques fail to extract a mask of the rings and crosses on this image as shown in the bottom left image. An opening of the original image with a large square SE removes all relevant image structures but preserves the

illumination function. The white top-hat of the original image or subtraction of the illumination function from the original image outputs an image with a homogeneous illumination. A global threshold of the white top-hat succeeds in extracting relevant image structures as shown in the bottom left image. The removal of the uneven illumination function of the watermark shown in table 1.1, p. 5, has been achieved with the same technique.

If the contrast between the objects and the background is decreasing when the background is darkening, a better visual rendering may be obtained by dividing the input image by the closing (or opening). This is illustrated in Fig. 4.19 for a very badly illuminated image of seeds. A closing with a large structuring element removes the seeds but preserves the illumination function. The black top-hat or subtraction of the original image from the closing provides us with an evenly illuminated image (Fig. 4.19c). A more contrasted image can be obtained by dividing the original image with its closing (Fig. 4.19d).
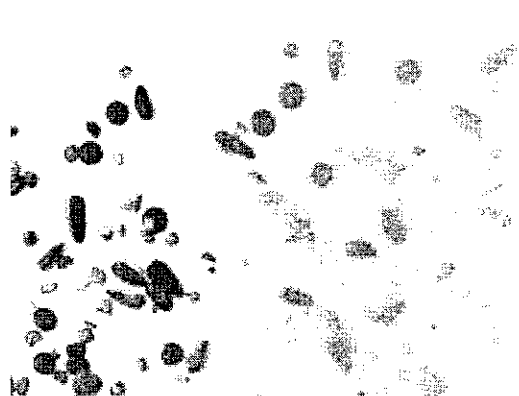

(a) Original image f (courtesy of Prof. B. Jähne, University of Heidelberg).
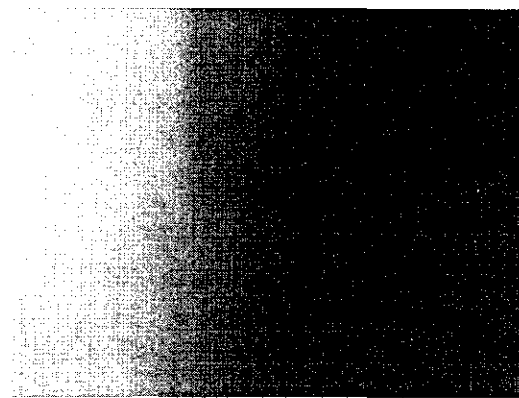
(b) Closing of f with a large square: $\phi(f)$
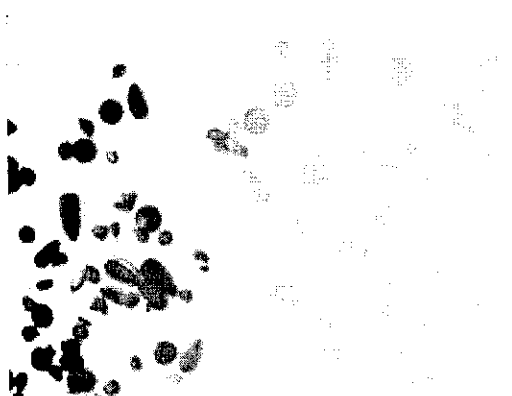
(c) Black top-hat: BTH$(f) = \phi(f) - f$.

(d) Division of f by $\phi(f)$.

Fig. 4.19. Use of top-hat for mitigating inhomogeneous illumination. In this example, a better rendering is obtained by dividing the original image by the closed image.

Note that in quality control applications where a series of objects are acquired at a fixed position, another solution consists in first capturing an

with the degree of reduction being determined by the g y
surrounding the bright detail and by the shape and amplitude values of the
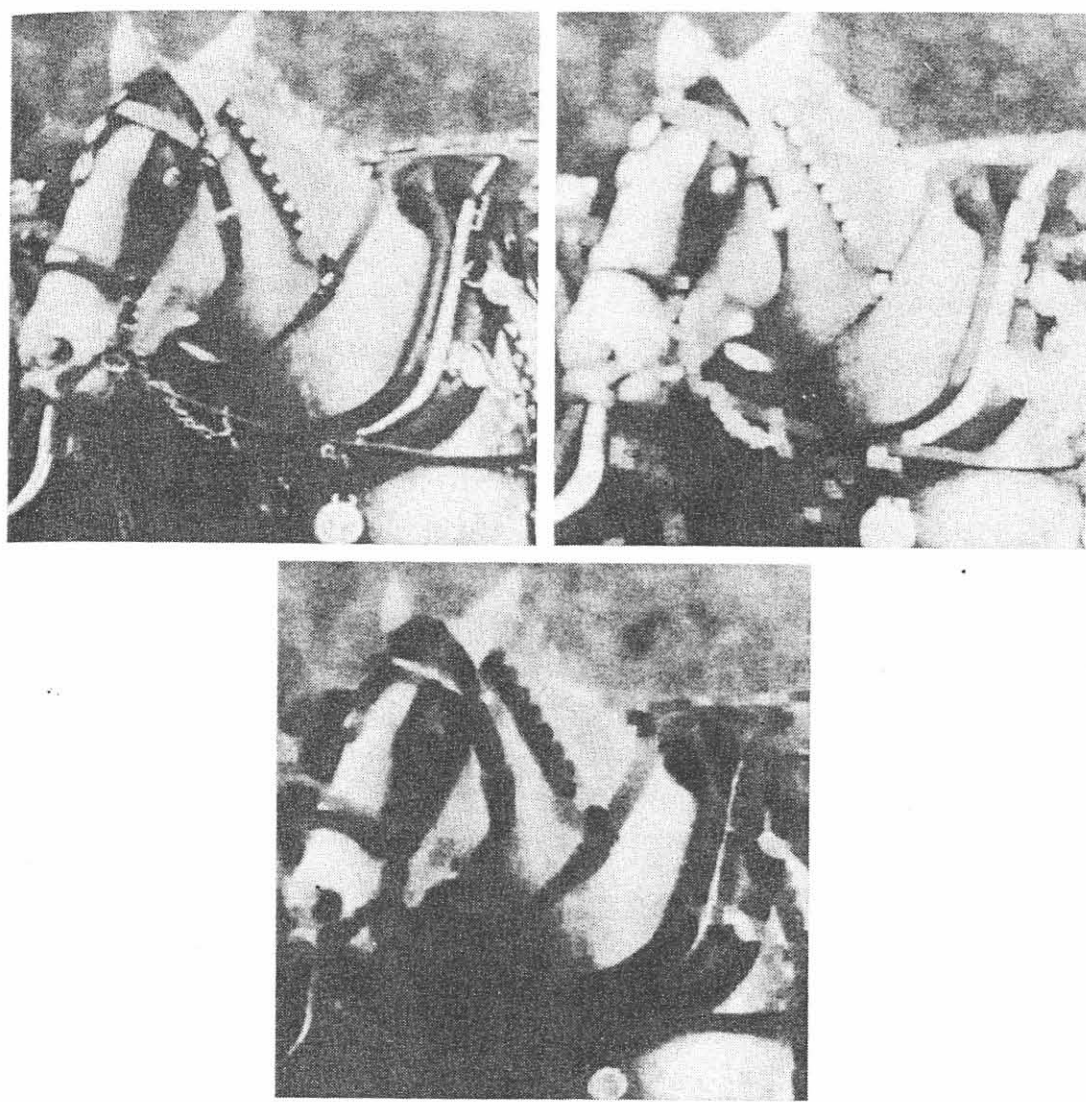structuring element itself.

Gray-scale dilation and erosion are duals with respect to function comple-
mentation and reflection. That is,

$$(f \ominus b)^c(s, t) = (f^c \oplus \hat{b})(s, t) \qquad (9.6\text{-}3)$$

where $f^c = -f(x, y)$ and $\hat{b} = b(-x, -y)$. Except as needed for clarity, we simplify
the notation in the following discussions by omitting the arguments of all functions.

■ Figure 9.29(a) shows a simple 512×512 gray-scale image, and Fig. 9.29(b)
shows the result of dilating this image with a "flat-top" structuring element in the
shape of a parallelepiped of unit height and size 5×5 pixels. Based on the pre-
ceding discussion, dilation is expected to produce an image that is brighter than
the original and in which small, dark details have been reduced or eliminated.

EXAMPLE 9.9:
Illustration of
dilation and
erosion on a gray
scale image.

a b
c

FIGURE 9.29
(a) Original
image. (b) Resul
of dilation.
(c) Result of
erosion.
(Courtesy of
Mr. A. Morris,
Leica Cambridg
Ltd.)