Averages

N^2 is number of pixels in the window

1. Arithmetic Mean $= \frac{1}{N^2} \sum_{r,c} d(r,c)$ low pass, blurs images

   reduces all noise, best for Gaussian & uniform

2. Geometric Mean: $= \left[\prod_{r,c} d(r,c) \right]^{\frac{1}{N^2}}$ best with Gaussian

   sharper than arithmetic, doesn't work on pepper

3. Harmonic mean: $= \frac{N^2}{\sum_{r,c} \frac{1}{d(r,c)}}$ good on salt - bad on pepper

4. Y_p mean $= \left[\sum_{r,c} \frac{d^p(r,c)}{N^2} \right]^{\frac{1}{p}}$ for $p > 0$ good on salt - bad on pepper

   for $p < 0$ bad on salt - good on pepper

5. Contra-harmonic mean $= \frac{\sum_{r,c} d^{R-1}(r,c)}{\sum_{r,c} d^R(r,c)}$ $R > 0$ eliminates salt

   $R < 0$ eliminates pepper

6. alpha - trim (T) $= \frac{1}{N^2-2T-1} \sum_{j=T+1}^{N^2-T} I_j$ between arithmetic and median

7. Nonlinear mean $= u^{-1} \left\{ \frac{\sum a_{ij} u[g_{ij}]}{\sum a_{ij}} \right\}$

   - Arithmetic mean $u(g) = g$
   - Harmonic mean $u(g) = \frac{1}{g}$
   - Geometric mean $u(g) = log(g)$

8. Minimum Mean-Square Error (MMSE)

$$MMSE = d(r,c) - \frac{\sigma_n^2}{\sigma_l^2}\left(d(r,c) - m_l(r,c)\right)$$

- σ_n^2 = noise variance
- σ_l^2 = local variance inside window
- $m_l(r,c)$=mean inside window

Bilateral Filter

Use some weighted average

$$\widehat{x}_k = \frac{\sum_n w_{kn} g_{k-n}}{\sum_n w_{kn}}$$

1. $w^s_{kn} = e^{-\frac{d^2_{k,k-n}}{2\sigma^2_s}}$ Gaussian, physical distance
2. $w^r_{kn} = e^{-\frac{(g_k - g_{k-n})^2}{2\sigma^2_r}}$ Gaussian, gray value distance

$$w_{kn} = w^s_{kn} w^r_{kn}$$

σ_s , σ_r large implies uniform non-adaptive , degrades signal

σ_s , σ_r small implies no smoothing

Weighted Least Squares

X=measured image
Y=desired image

$$\varepsilon_{\mathrm{WLS}} = \frac{1}{2}\left(X-Y\right)^T\left(X-Y\right) + \underbrace{\frac{\lambda}{2}L(X)^T W(Y) L(X)}_{\text{penalty}}$$

L(X) can be first or second derivative

Robust Estmation:

$$\varepsilon_{\mathrm{WLS}} = \frac{1}{2}\left(X-Y\right)^T\left(X-Y\right) + \frac{\lambda}{2}\underbrace{\rho(X)}_{\text{nonlinear}}$$