Examples

Definition: Linear $au_1 + bu_2$ is also a solution

- semi-linear, quasi-linear
- $\frac{\partial u}{\partial x} + a\frac{\partial u}{\partial y} = 0 \qquad \frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$
- $\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0 \qquad \frac{\partial u}{\partial t} + x\frac{\partial u}{\partial x} = 0$
- $\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y} = 0 \qquad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$
- $\triangle u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y)$ Laplace/Poisson
- $x\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ Tricomi
- $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + k^2 u = f(x,y)$ Helmholtz
- $\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0$ Wave
- $\frac{\partial u}{\partial t} - \kappa\frac{\partial^2 u}{\partial x^2} = 0$ heat/diffusion $\kappa > 0$
- $\frac{\partial^2 u}{\partial t^2} + \kappa\frac{\partial^4 u}{\partial x^4} = 0$ vibrating bar $\kappa > 0$
- $\frac{\partial u}{\partial t} - i\frac{\partial^2 u}{\partial x^2} = 0$ Schroedinger
- $(2\alpha + \beta)\frac{\partial^2 H}{\partial x^2} + (\alpha + 2\beta)\frac{\partial^2 H}{\partial y^2} = 0$ glass manufacture $u = \alpha x \qquad v = \beta y$
- $u = \frac{\partial \varphi}{\partial x} \qquad v = \frac{\partial \varphi}{\partial y} \qquad (a^2 - u^2)\frac{\partial^2 \varphi}{\partial x^2} - 2uv\frac{\partial^2 \varphi}{\partial x \partial y} + (a^2 - v^2)\frac{\partial^2 \varphi}{\partial y^2} = 0$ potential
- System
  $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$
  $\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0$ Cauchy-Riemann
- $\frac{\partial u}{\partial t} - c\frac{\partial v}{\partial x} = 0$
  $\frac{\partial v}{\partial t} - c\frac{\partial u}{\partial x} = 0$
- $\frac{\partial p}{\partial t} - \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0$
  $\frac{\partial u}{\partial t} - \frac{\partial p}{\partial x} = 0$
  $\frac{\partial v}{\partial t} - \frac{\partial p}{\partial y} = 0$ acoustics
- $\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + \gamma p(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) = 0$
  $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$
  $\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial y} = 0$ Euler

- $\frac{\partial B}{\partial t} + c \quad curl(E) = 0$
  $\frac{\partial E}{\partial t} - c \quad curl(B) = -4\pi J$ Maxwell

Well-posedness

- existence
- uniqueness
- continuous dependence