Non-Intrusive Medical Diagnosis

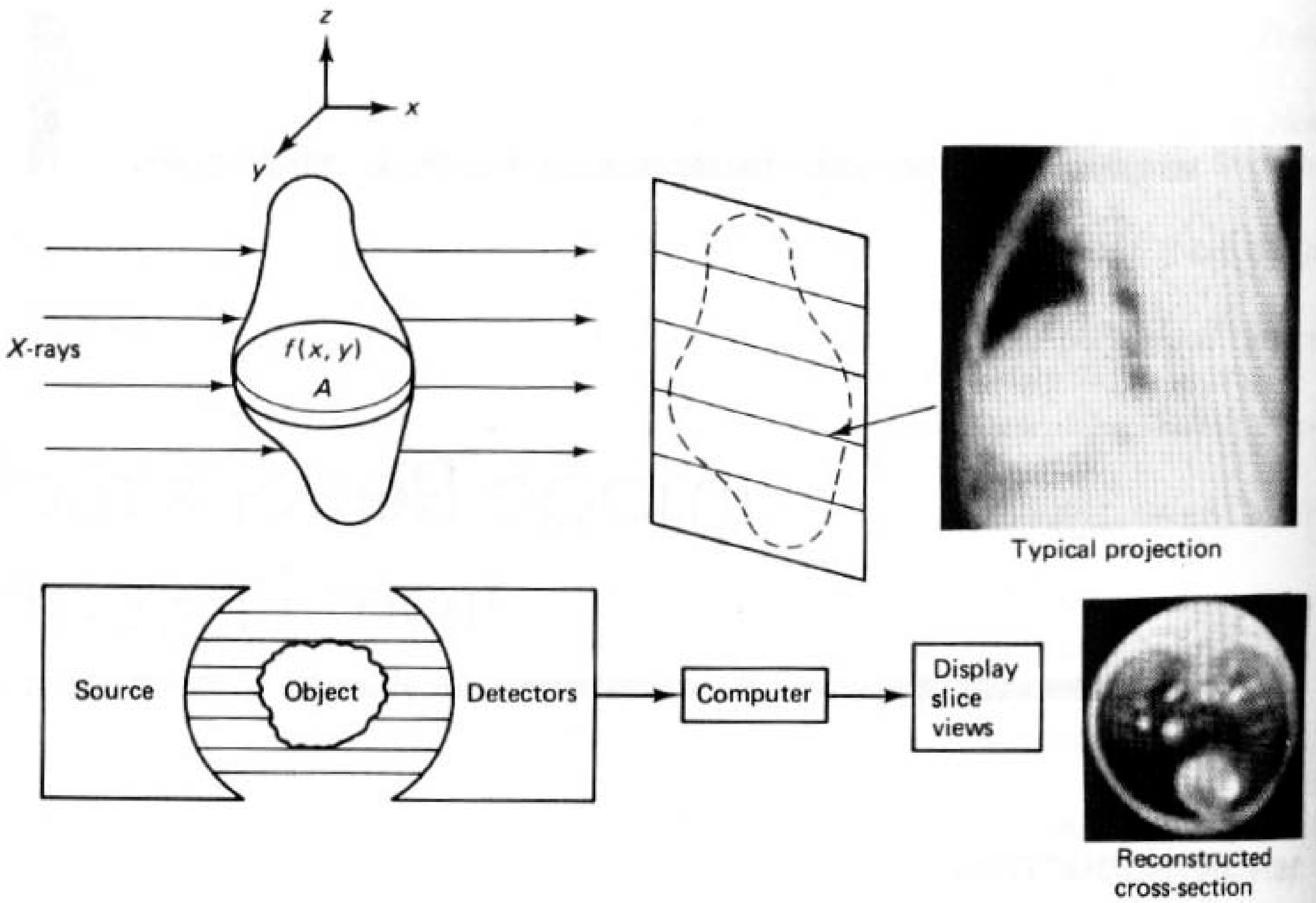
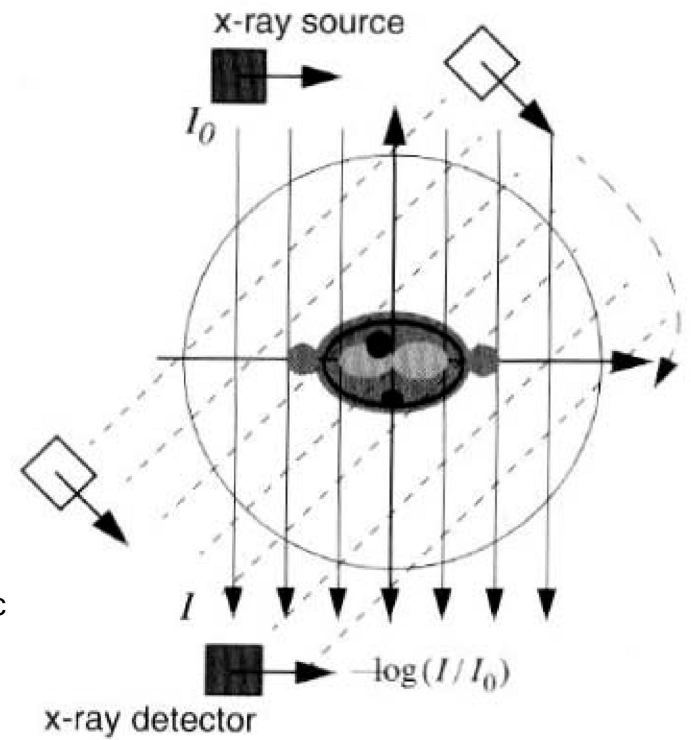


Figure 10.1 An X-ray CT scanning system.

Non-Intrusive Medical Diagnosis (cont'd)

- Observe a set of projections (integrations) along different angles of a cross-section
  - Each projection itself loses the resolution of inner structure
  - Types of measurements
    - *transmission (X-ray), emission, magnetic resonance (MRI)*
- Want to recover inner structure from the projections
  - "Computerized Tomography" (CT)



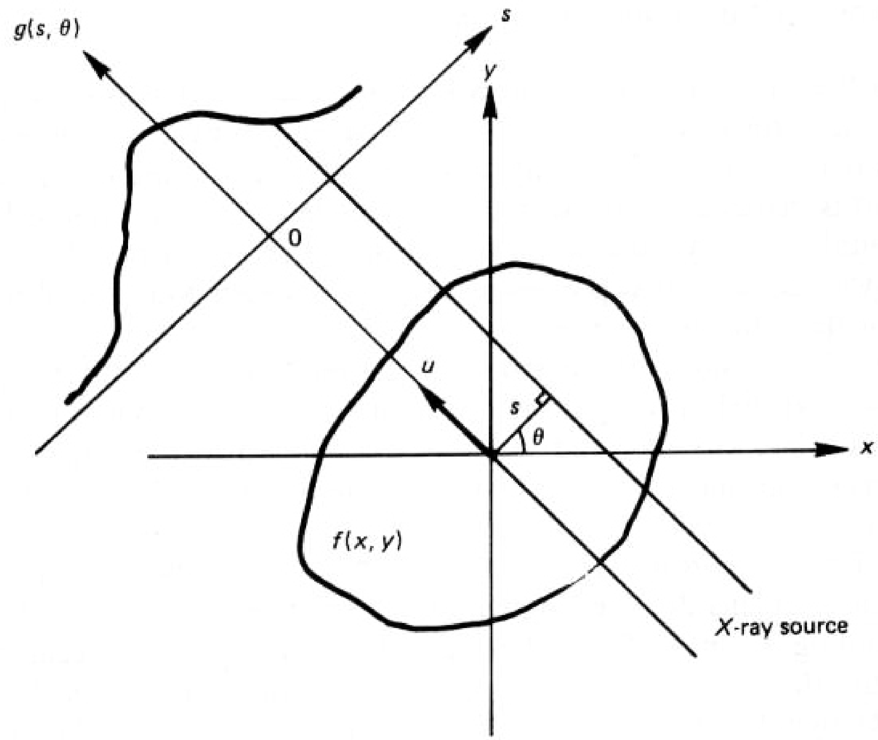
(From Bovik's Handbook Fig.10.2.1)

Emission tomography: measure emitted gamma rays by the decay of isotopes from radioactive nuclei of certain chemical compounds affixed to body parts.

MRI: based on that protons possess a magnetic moment and spin. In magnetic field => align to parallel or antiparallel. Apply RF => align to antiparallel. Remove RF => absorbed energy is remitted and detected by Rfdetector.

Radon Transform

- A linear transform
  f(x,y) → g(s,θ)
  - Line integral or "ray-sum"
  - Along a line inclined at angle θ from y-axis and s away from origin
- Fix θ to get a 1-D signal $g_\theta(s)$



Projection imaging geometry in CT scanning.

(From Jain's Fig.10.2)

$$g(s,\theta) = \iint_{-\infty}^{+\infty} f(x,y)\delta(x\cos\theta + y\sin\theta - s)\,dx\,dy$$

$$= \int_{-\infty}^{+\infty} f(s\cos\theta - u\sin\theta, s\sin\theta + u\cos\theta)\,du$$

$$\text{where} \begin{bmatrix} s \\ u \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \text{(coordinate rotation)}$$

Example of Image Radon Transform

[Y-axis] distance, [X-axis] angle

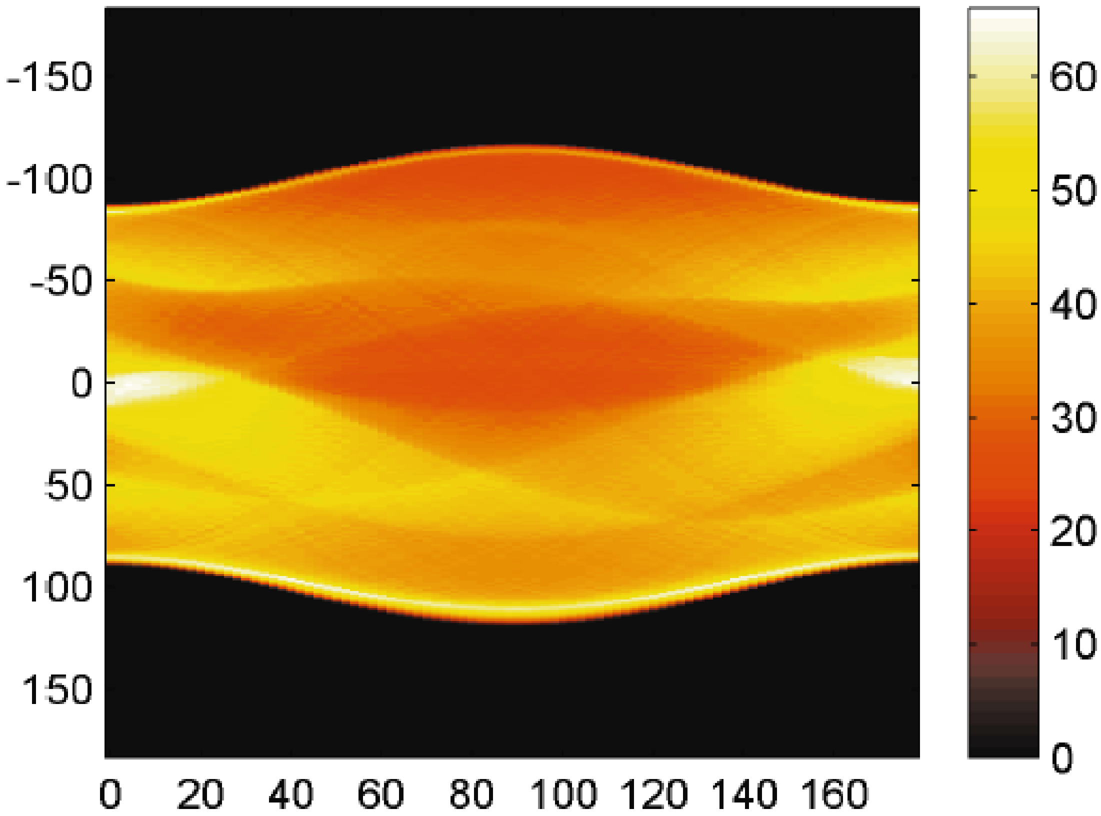
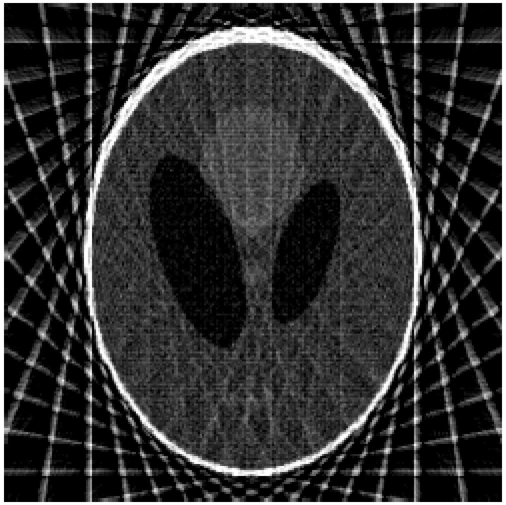


Figure 8-18: Radon Transform of Head Phantom Using 90 Projections

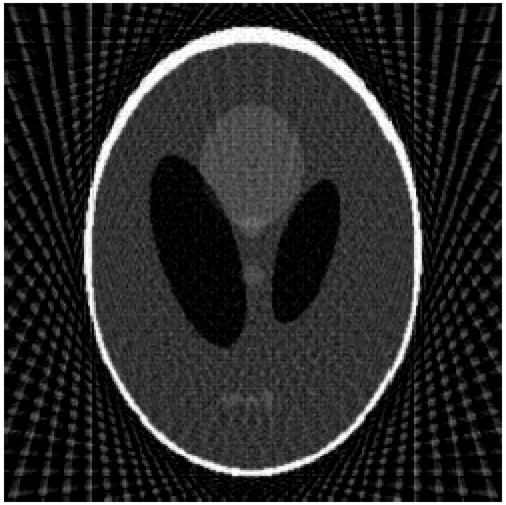
(From Matlab Image Processing Toolbox Documentation)

Inverting A Radon Transform

- To recover inner structure from projections
- Need many projections to better recover the inner structure



I1



I2

I3

Reconstruction from 18, 36, and 90 projections (~ every 10,5,2 degrees)

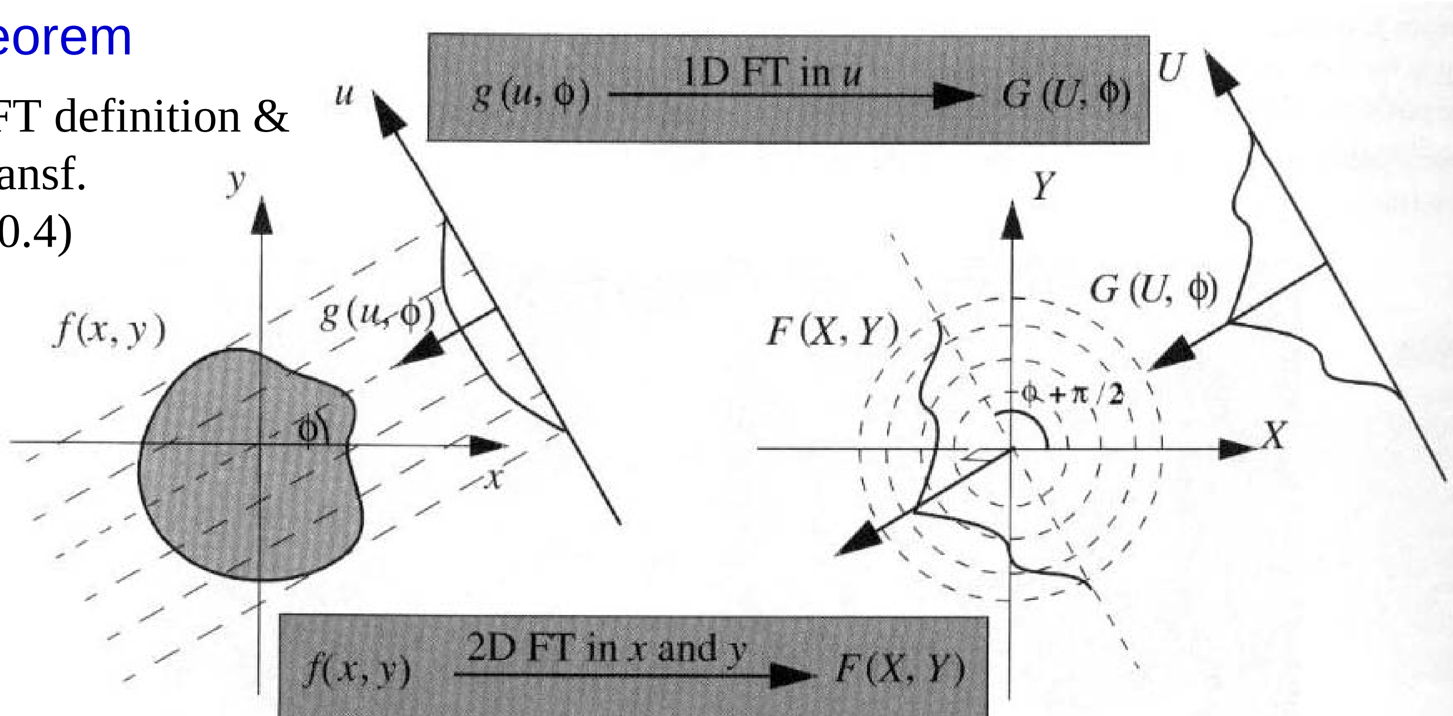
(From Matlab Image Processing Toolbox Documentation)

Connection Between Radon & Fourier Transf.

- Observations
  - Look at 2-D FT coeff. along horizontal frequency axis
    - *FT of 1-D signal*
    - *1-D signal is vertical summation (projection) of original 2-D signal*
  - Look at FT coeff. along $\theta = \theta_0$ ray passing origin
    - *FT of projection of the signal perpendicular to* $\theta = \theta_0$

(From Bovik's Handbook Fig.10.2.7)

- Projection Theorem
  - Proof using FT definition & coordinate transf. (Jain's Sec.10.4)



Inverting Radon by Projection Theorem

- (Step-1) Filling 2-D FT with 1-D FT of Radon along different angles
- (Step-2) 2-D IFT
- Need Polar-to-Cartesian grid conversion for discrete scenarios
  - May lead to artifacts

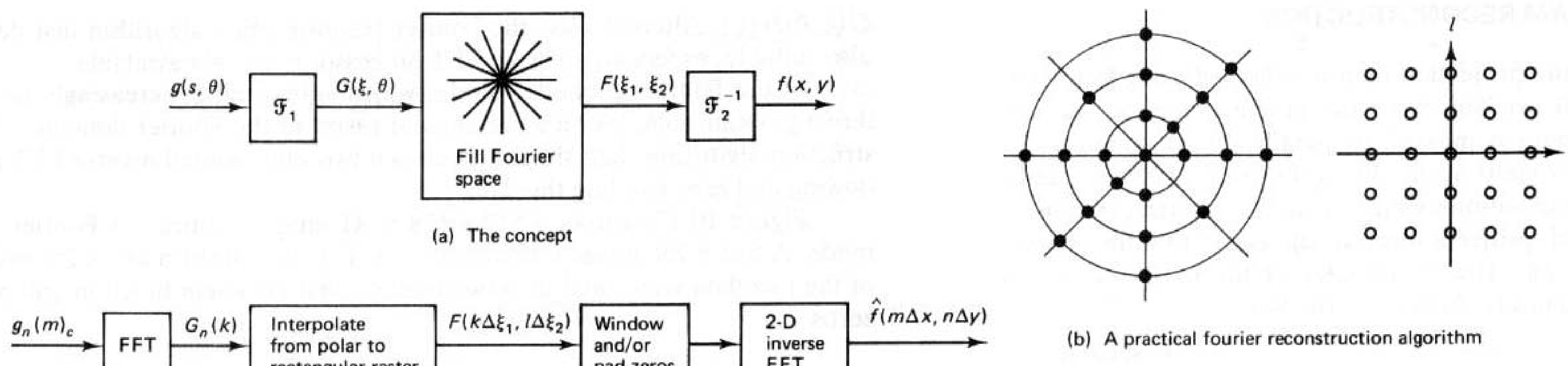
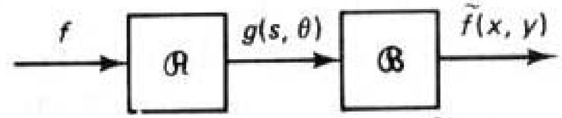


Figure 10.16 Fourier reconstruction method.

(From Jain's Fig.10.16)

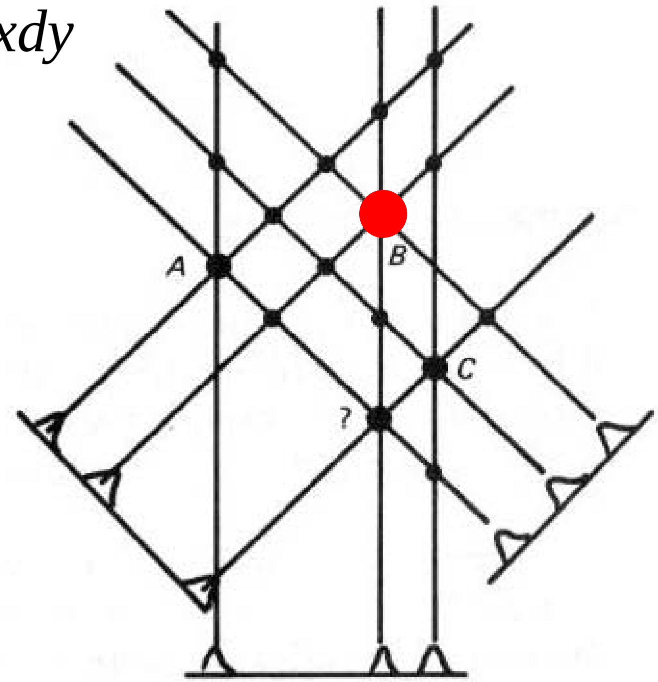
Back-Projection

- Sum up Radon projection along all angles passing the same pixels



$$g(s,\theta) = \int\int_{-\infty}^{+\infty} f(x,y)\delta(x\cos\theta + y\sin\theta - s)\,dx\,dy$$

$$\tilde{f}(x,y) = \int_0^{\pi} g(x\cos\theta + y\sin\theta, \theta)\,d\theta$$



Back-projected projections $\tilde{f}(x, y)$

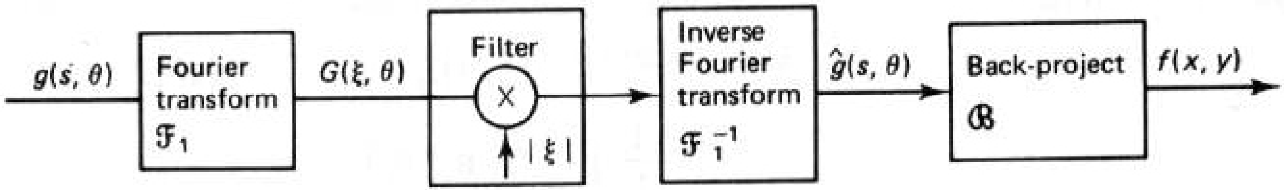
(From Jain's Fig.10.6)

Back-projection = Inverse Radon ?

- Not exactly ~ Back-projection gives a blurred recovery
  - $\mathcal{B}(\mathcal{R} f) = \text{conv}(f, h1)$
  - Bluring func. $h1 = (x^2 + y^2)^{-1/2}$, FT(h1) ~ $1 / |\xi|$ where $\xi^2 = \xi_x^2 + \xi_y^2$
  - Intuition: most contribution is from the pixel (x,y), but still has some tiny contribution from other pixels

- Need to apply inverse filtering to fully recover the original
  - Inverse filter for "sharpening"
    - *multiplied by $|\xi|$ in FT domain*

Inverting Radon via Filtered Back Projection



(c) Filter back-projection method

Figure 10.8 Inverse radon transform methods.

(From Jain's Fig.10.8)

f(x,y) = $\mathcal{B}\,\mathcal{H}$ g

$$f(x,y) = \iint_{-\infty}^{+\infty} F(\xi_x, \xi_y) \exp[j2\pi(\xi_x x + \xi_y y)] d\xi_x d\xi_y$$

$$= \int_0^{2\pi} \int_0^{\infty} F_{polar}(\xi, \theta) \exp[j2\pi\xi(x\cos\theta + y\sin\theta)] \xi d\xi d\theta$$

Change coordinate (Cartesian => polar)

$$= \int_0^{\pi} \int_{-\infty}^{\infty} |\xi| F_{polar}(\xi, \theta) \exp[j2\pi\xi(x\cos\theta + y\sin\theta)] d\xi d\theta$$

$$= \int_0^{\pi} \left\{ \int_{-\infty}^{\infty} |\xi| G(\xi, \theta) \exp[j2\pi\xi(x\cos\theta + y\sin\theta)] d\xi \right\} d\theta$$

Projection Theorem (F_polar => G)

$$= \int_0^{\pi} \hat{g}(x\cos\theta + y\sin\theta, \theta) d\theta \quad \text{where } \hat{g}(s,\theta) = \int_{-\infty}^{\infty} |\xi| G(\xi, \theta) e^{j2\pi s\xi} d\xi$$

Back Projection

(FT domain) filtering

Filtered Back Projection (cont'd)

- Convolution-Projection Theorem
  - Radon[f1 (*) f2] = Radon[f1] (*) Radon[f2]
    - *Radon and filtering operations are interchangeable*
    - *can prove using Projection Theorem*
  - Also useful for implementing 2-D filtering using 1-D filtering
- Another view of filtered back projection
  - Change the order of filtering and back-proj.
    - *Back Projection => Filtering*
    - *Filtering => Back Projection*

Other Scenarios of Computerized Tomography

- Parallel beams vs. Fan beams
  - Faster collection of projections via fan beams
    - *involve rotations only*

(From Bovik's Handbook Fig.10.2.1)

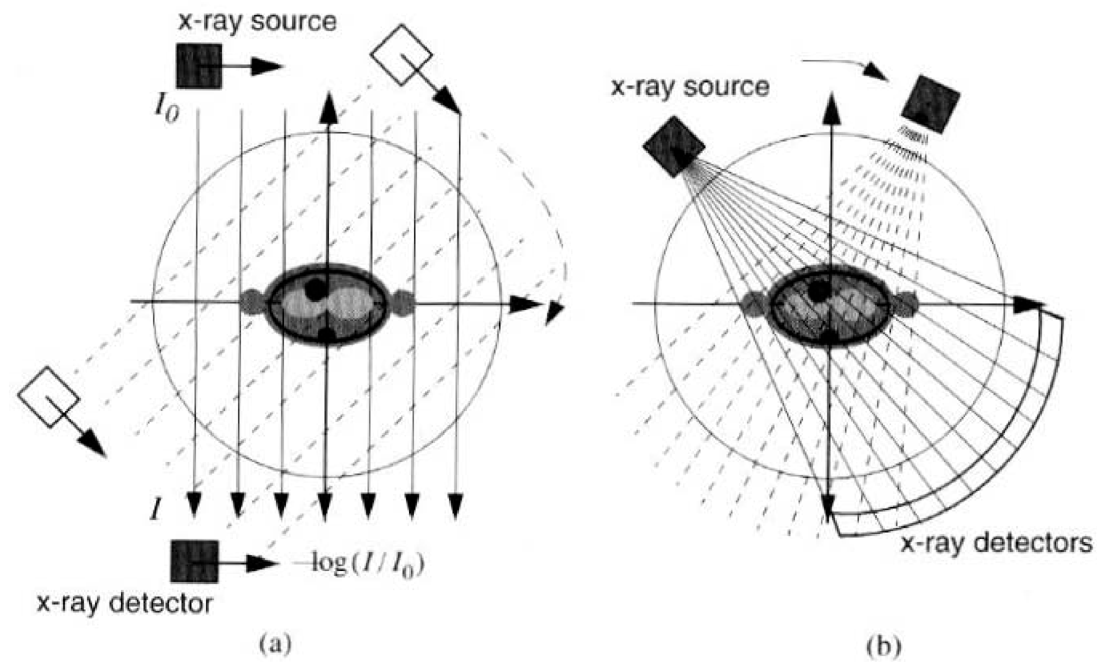


FIGURE 1 (a) Schematic representation of a first-generation CT scanner that uses translation and rotation of the source and a single detector to collect a complete set of 1-D parallel projections. (b) The current generation of CT scanners uses a fan X-ray beam and an array of detectors, which require rotation only. (From Bovik's Handbook Fig.10.2.1)

- Recover from projections contaminated with noise
  - MMSE criterion to minimize reconstruction errors

➔ See Jain's book and Bovik's Handbook for details

Summary

- Medical Imaging Topic
  - Radon transform
  - Inverse Radon transform
    - *by Projection Theorem*
    - *by filtered back-projection*

- 2^{nd} Review

Summary of Lecture 11 ~ 21

Overview

- Digital Video Processing
  - Basics
  - Motion compensation
  - Hybrid video coding and standards
  - Brief intro. on a few advanced topics ~ *object-based, content analysis, etc.*
  - Interpolation problems for video
    - *sampling lattice*
- Image Manipulation / Enhancement / Restoration
  - Pixel-wise operations
  - Coefficient-wise operations in transform domain
  - Filtering: FIR, nonlinear, Wiener, edge detection, interpolation
  - Geometrical manipulations: RST, reflection, warping
  - Morphological operations on binary images

Video Formats, etc.

- Video signal as a 3-D signal
- FT analysis and freq. response of HVS
- Video capturing and display
- Analog video format
- Digital video format

Motion Estimation

- 3-D and projected 2-D motion models
- Optical Flow Equation for estimating motion
- General approaches of motion compensation & key issues
- Block-Matching Algorithms
  - Exhaustive search
  - Fast algorithms
  - Pros and Cons
- Other motion estimation algorithms – basic ideas

Hybrid Video Coding and Standards

- Transf. Coding + Predictive Coding
- Key points of MPEG-1
- Scalability provided in MPEG-2
- Object-based coding idea in MPEG-4

Pixel-wise Operations for Enhancement

- Specified by Input-Output luminance or color mapping
- Commonly used operations
  - Contrast stretching
  - Histogram equalization

Simple Filters of Finite-Support

- Convolve an image with a 2-D filter of finite support
- Commonly used FIR filters
  - Averaging and other LPFs for noise reduction
  - Use LPF to construct HPF and BPF
    - *for image sharpening*
- Nonlinear filtering
  - Median filter ~ remove salt-and-pepper noise
- Edge Detection
  - Estimate gradient of luminance or color
    - *Equiv. to directional HPF or BPF*
  - Common edge detectors

Wiener Filtering

- Inverse filtering and pseudo-inverse filtering
  - De-blurring applications
- Wiener filtering for restoration in presence of noise
  - MMSE criterion
  - Orthogonal principle
  - Wiener filter (in terms of auto/cross-correlation and PSD)
  - Relations of Wiener filter with inverse and pseudo-inverse filters
- Basic ideas of blind deconvolutions

Interpolation

- 1-D sampling rate conversion
  - Ideal approach and frequency-domain interpretation
  - Practical interpolation approaches
- 2-D interpolation for rectangular sampling lattice
  - Ideal approach and practical approaches
- Sampling lattice conversion
  - Basic concepts on sampling lattice
  - Ideal approach for sampling lattice conversion
  - Applications in video format conversion
    - *practical approaches and their pros & cons*

Geometrically Manipulations

- Rotation, Scale, Translation, and Reflection
  - Homogeneous coordinates
  - Interpolation issues in implementation: forward v.s. backward transform
- Polynomial warping
- Line-based warping and image morphing