

# THE STATE OF GEOMETRY AND FUNCTIONAL ANALYSIS

June 24–30, 2009

## TITLES AND ABSTRACTS

**Wednesday, June 24**

**Morning session chairperson: N. Tomczak-Jaegermann**

9:45 **H. Wolfson**, Dean, Faculty of Exact Sciences – *Greetings to the conference and opening remarks*

10:00–10:50 **N. Alon** – *Graph Eigenvalues and the interplay between discrete and continuous isoperimetric problems*

**Abstract:** The spectral properties of a graph are intimately related to its structural properties. This can be applied in the study of discrete isoperimetric problems and in the investigation of extremal and algorithmic questions for graphs, revealing an intriguing connection between discrete and continuous problems and techniques.

11:00–11:50 **Y. Eliashberg** – Leonard M. Blumenthal Lecture in Geometry – *Symplectic topology of Stein manifolds*

**Abstract:** Morse theory of plurisubharmonic functions on a Stein manifold  $X$  is governed by its symplectic topology. In the talk I will define and explain how to compute certain symplectic invariants of  $X$  (e.g. its symplectic homology) in terms of a symplectic handle decomposition of  $X$ , given by an exhausting plurisubharmonic Morse function, or in terms of a presentation of  $X$  as a symplectic Lefschetz fibration. When complex dimension of  $X$  is equal to 2, the answer provides a purely combinatorial description of the corresponding complex. This is a joint work with F. Bourgeois and T. Ekholm.

**Afternoon session chairperson: P. Gruber**

13:50–14:40 **G.S. Bobkov** – *Geometric and analytic problems in the theory of heavy-tailed convex measures*

**Abstract:** We give a short overview of some results on high-dimensional convex measures with heavy tails, obtained in the recent years. They include a number of geometric inequalities of dilation-type, isoperimetric and weighted Poincaré and Cheeger inequalities. Different approaches to the isotropicity are also addressed in the light of Hensley's theorem on slices of convex bodies.

14:50–15:40 **P. Enflo** – *Extremal vectors and invariant subspaces*

**Abstract.** In some Banach spaces, there are continuous linear operators with only trivial closed invariant subspaces. However, the methods to construct such operators have, so far, only worked in certain non-reflexive Banach spaces. The problem, whether every continuous linear operator has a non-trivial invariant subspace is still open in reflexive spaces, and in particular, in Hilbert space. Existence of non-trivial invariant subspaces has been proved for many classes of operators on Hilbert space. In 1995 the author discovered that various types of extremal vectors can be used to construct invariant subspaces for different classes of operators on Hilbert space. One example of extremal vector is the best approximate solution  $x$  of the (unsolvable) equation  $Tu'x = y$ , when  $T$  is an operator with dense range on Hilbert space and  $y$  is not in the range of  $T$ . This approach to the invariant subspace problem for Hilbert space leads to new types of problems on operators on Hilbert space, problems which may be of interest in themselves.

NEW TIMES

16:00–16:50 **A. Wigderson** – *Applications of TCS techniques to geometric problems*

17:15 *Ceremony: Honorary Doctorate award to Professor Mikhail Gromov*

17:30–18:20 **M. Gromov** – *TBA*

21:30 Buses leave Tel Aviv University for the hotels, main gate

**Thursday, June 25**

**Morning session chairperson: A. Zvavitch**

9:30–10:20 **D. Kazhdan** – *Satake isomorphism for symmetrizable Kac-Moody algebras*

10:40–11:40 **J. Lindenstrauss** – Frechet differentiability, porosity and asymptotic structure in Banach spaces

**Abstract:** The modulus of asymptotic uniform smoothness of a Banach space is defined by

$$\bar{\rho}_X(t) = \sup_{\|x\|=1} \inf_{\dim X/Y < \infty} \sup_{\substack{y \in Y \\ \|y\| \leq t}} (\|x + y\| - 1)$$

There is also a dual notion of asymptotic uniform convexity. With suitable assumptions on these moduli several results on the existence of points of Fréchet differentiability of Lipschitz functions from a Banach space  $X$  to the reals  $R$  (and even into a general Banach space with RNP) are proved. Closely related to these results are results on the structure of porous sets in  $X$ . One of the tools used in the proof is a new variational principle on not necessarily complete metric spaces. Most of the proofs are very long and involve hard analysis in  $R^n$ . This talk is based on a joint book with D. Preiss and J. Tišer which is now being written.

11:40–12:30 **S. Argyros** – *The solution of the “Scalar plus Compact” problem*

**Afternoon session chairperson: D. Ryabogin**

14:00–14:50 **I. Benjamini** – *Random planar geometry*

**Abstract:** We will survey aspects of random planar metrics and discuss some related variants.

15:00–15:50 **J. Bourgain** – *Expansion in linear groups and applications*

**Abstract:** This is a review of recent work on spectral gaps of Hecke operators on linear groups, obtained in collaboration with various coauthors. The general theme is a conjecture of Lubotzky and Weiss, stating that if  $S$  is a finite subset of  $SL^d(Z)$  generating a subgroup Zariski dense in  $SL^d$ , then reduction mod  $q$  gives an expander family  $(SL^d(q), S)$ , provided  $q$  relative prime with some integer  $q(S)$ . Various cases can now be handled and in particular the case  $d = 2$  is pretty much understood. Also similar results in the unitary setting are obtained. An important ingredient in the approach are techniques from arithmetic combinatorics. There are many consequences but we will mainly focus on hyperbolic lattice point counting and prime sieving in orbits of linear groups in the talk.

16:30–18:30 Mini-symposium in *Asymptotic Geometric Analysis*

Organizers: A. Litvak and V. Milman

Participants:

**J. Bernues** – *On the isotropy constant of projections of polytopes* (joint work with D. Alonso, J. Bastero, and P. Wolff)

**M. Fradelizi** – *Lower bound for the volume product of convex bodies with the symmetries of a regular polyhedron* (joint work with F. Barthe)

**B. Kashin** – *Orthomassivity as a measure of complexity*

**O. Maleva** – *Null sets which are not Lipschitz negligible*

**T. Schlumprecht** – *The universality of  $\ell_1$  as dual space*

**S. Szarek** – *Between the real and the complex*

**A. Tsolomitis** – *Asymptotic shape of a random polytope in a convex body* (joint work with N. Dafnis and A. Giannopoulos)

19:00 Buses leave Tel Aviv University for hotels, gate 2

**Friday, June 26 at the Dead Sea**  
**Morning session chairperson: A. Litvak**

11:00–11:50 **L. Pastur** - *Universalities in random matrix theory*

**Abstract:** We present a brief review of a collection of results (including recent) on eigenvalue distribution of random matrices, paying special attention to similarity (even coincidence) of results for different classes of ensembles of random matrices.

**Afternoon session chairperson: I. Bárány**

14:30–15:20 **A. Giannopoulos** - *Volume distribution on high dimensional convex bodies*

**Abstract:** In this survey talk we describe recent developments (mainly due to Bo'az Klartag and Grigoris Paouris) related to the distribution of volume on isotropic convex bodies. A convex body  $K$  in  $\mathbb{R}^n$  is called isotropic if it has volume one, center of mass at the origin, and there exists a constant  $L_K > 0$  such that

$$\int_K \langle y, \theta \rangle^2 dy = L_K^2$$

for every  $\theta \in S^{n-1}$ . Our starting point is the well-known slicing problem which asks if there exists an absolute constant  $C > 0$  such that  $L_K \leq C$  for every isotropic convex body in any dimension. We discuss recent results on sharp dimension-dependent concentration of volume, existence of “subgaussian directions”, stability of moments of the Euclidean norm, shape and volume of random polytopes spanned by a fixed number of points from the body, and several related questions. We focus on the work of Paouris on the family of the  $L_q$ -centroid bodies of an isotropic convex body  $K$ . The behavior of this family of bodies as  $q$  grows to  $n$  is closely related to all these questions.

15:30–16:20 **D. Preiss** - *Lipschitz functions and negligible sets*

The talk will concentrate on a strange phenomenon that various people noticed when studying diverse geometric/measure theoretic problems in Euclidean spaces, such as: Can positive measures uniquely determine directions of naturally associated vector-valued measures? Can sets of positive Lebesgue measure be mapped onto balls? Can Rademacher’s theorem on differentiability of Lipschitz functions be improved? The talk will be based on a joint work with G. Alberti and M. Csornyei, and also try to put into this context relevant results of M. Dore and O. Maleva as well as some related result of a joint work with J. Lindenstrauss and J. Tiser.

16:30–18:30 Mini symposium in *Classical Convexity*

Organizers: S. Alesker and M. Ludwig

Participants:

**Imre Bárány** - *Limit shape results for convex polygons*

**Abstract:** Consider the lattice  $t\mathbb{Z}^2$  in the plane where  $t > 0$  is small, and the family  $F_t$  of all convex  $t\mathbb{Z}^2$ -lattice polygons contained in the unit square. The family  $F_t$  is finite, and, as  $t$  tends to zero, the overwhelming majority of the polygons in  $F_t$  are very close to a unique convex set  $K$ , which is the “limit shape” of the collection  $F$ . I will try to explain, in the given time-frame, how and why the same limit shape phenomenon appears in this case and in other problems as well.

**Andreas Bernig** - *Algebraic integral geometry*

**Abstract:** I will present some results on algebraic structures on the space of translation invariant valuations and give some applications in integral geometry.

**Károly J. Böröczky** - *Stability of the Blaschke–Santaló and the affine isoperimetric inequality*

**Abstract:** A stability version of the Blaschke–Santaló inequality and the affine isoperimetric inequality for convex bodies of dimension  $n > 2$  is presented. In addition related (possible) versions of the Prekopa–Leindler inequality are discussed.

**Daniel Hug** – *Stability of the inverse Blaschke–Santaló inequality for zonoids and applications*

**Abstract:** An important  $GL(n)$  invariant functional of centred convex bodies is the volume product. For a centred convex body  $A \subset \mathbb{R}^n$  it is defined by  $P(A) := |A| \cdot |A^*|$ , where  $|\cdot|$  denotes volume and  $A^*$  is the polar body of  $A$ . A sharp upper bound for  $P$  is provided by the famous Blaschke–Santaló inequality. A major open problem asks for a sharp lower bound for  $P$ . If  $A$  is a centred zonoid, then it is known that  $P(A) \geq P(P_\infty^n)$ , where  $P_\infty^n$  is a centred parallelotope (Minkowski sum of linearly independent segments). Equality holds in the class of zonoids if and only if  $A$  is a centred parallelotope. Here we sharpen this uniqueness statement in terms of a stability result by showing that the Banach–Mazur distance of a centred zonoid  $A$  from a centred parallelotope is small if  $P(A) \leq (1 + \epsilon) \cdot P(P_\infty^n)$ ,  $\epsilon > 0$ . This result is then applied to strengthen a uniqueness result in stochastic geometry.

**Alex Koldobsky** – *Positive definite functions and multidimensional versions of random variables*

**Abstract:** We prove that if a function of the form  $f(\|\cdot\|_K)$  is positive definite on  $\mathbb{R}^n$ , where  $K$  is an origin symmetric star body in  $\mathbb{R}^n$  and  $f: \mathbb{R} \rightarrow \mathbb{R}$  is an even continuous function, then the space  $(\mathbb{R}^n, \|\cdot\|_K)$  embeds in  $L_0$ . We apply this result to the problem of characterization of  $n$ -dimensional versions, i.e. random vectors in  $\mathbb{R}^n$  such that all linear combinations of their coordinates have equal distribution, up to a scalar constant. Combined with known facts about embedding in  $L_0$ , this result also leads to several generalizations of the solution of Schoenberg’s problem on positive definite functions.

**Dmitry Ryabogin** – *On the Mahler conjecture: local minimality of the unit cube*

**Abstract:** This is a joint work with Fedor Nazarov, Fedor Petrov and Artem Zvavitch. We prove that the unit cube is a strict local minimizer for the Mahler volume product in the class of origin-symmetric convex bodies endowed with the Banach–Mazur distance.

**Alina Stancu** – *On some affine inequalities for smooth convex bodies*

**Abstract:** We will present a sharp affine inequality for an affine invariant of a smooth convex body containing the origin and with everywhere positive Gauss curvature followed by some consequences.

**Wolfgang Weil** – *Integral geometry of translation invariant functionals*

**Abstract:** For the classical intrinsic volumes of convex bodies and their local counterparts, the curvature measures, translative integral formulas are known. We show that such formulas also hold for general translation invariant continuous valuations. Even more, the valuation property is not essential here, if we require instead that the functional has a local counterpart. We give integral formulas for such local functionals and discuss the relation between local functionals and valuations.

**Elisabeth Werner** – *On the homothety conjecture* (by Elisabeth Werner and Deping Ye)

**Abstract:** The homothety conjecture asks: If a convex body  $K$  is homothetic to one of its floating bodies, is then  $K$  an ellipsoid? We prove that the homothety conjecture holds true in the class of the convex bodies  $B_p^n$ ,  $1 \leq p \leq \infty$ , the unit balls of  $l_p^n$ ; namely, we show that  $(B_p^n)_\delta = cB_p^n$  if and only if  $p = 2$ . We also show that the homothety conjecture holds true for small enough  $\delta$ , thus improving on earlier results by Schütt and Werner and Stancu.

**Saturday, June 27 – Day of Rest**

**Sunday, June 28**

**Chairperson: R. Schneider**

9:00–9:50 **P. Gruber** – *A short survey of the geometry of numbers and modern applications of a classical idea*

**Abstract:** The main branches of the geometry of numbers are lattice packing, lattice covering, lattice tiling, quadratic forms and special problems. After a short outline of these areas, applications of an idea of Voronoi to refined extremum properties of lattice packings of balls and smooth convex bodies, the Epstein zeta function and a generalization of it, and to John type and minimum position problems are presented.

10:00–10:50 **M. Ledoux:** – *Measure concentration, functional inequalities, and curvature of metric measure spaces*

**Abstract:** We survey recent developments on the geometry of metric measure spaces. Together with mass transportation methods and evolution equations, we emphasize in particular the role of (dimension free) measure concentration and functional inequalities in the emergence of an intrinsic definition of a lower bound on Ricci curvature in metric measure spaces.

11:30–12:30 **N. Tomczak-Jaegermann** – *Random matrices with independent columns*

**Monday, June 29**

**Morning session chairperson: G. Kalai**

9:30–10:20 **O. Zeitouni** – *Limit laws for the eigenvalues of certain matrix ensembles*

**Abstract:** Recent spectacular advance has been made in the study of the singular values and spectrum of (non-Hermitian) random matrices with iid entries, through the work of Rudelson and Vershynin, Tao and Vu, Gotze and Tikhomirov, and Erdos, Schlein and Yau. Less is known in models with dependent entries. I will describe some recent progress concerning the spectrum of non-Hermitian matrices satisfying an appropriate invariance in law requirement. Concentration with respect to Haar measure will play an important role.

10:40 - 11:40 **B. Bollobás** – *Convergent sequences of graphs*

**Abstract:** What are sensible metrics on the space of all finite graphs? Given an appropriate metric, is there a natural way to complete the space? What information is carried by a limit object? Questions like these have been investigated by several authors, including the two independent groups of Borgs, Chayes, Lovász, Sós, Szegedy and Vesztergombi, and Bollobás, Janson and Riordan. In this talk we shall review some of the main results obtained so far, and shall present a number of open problems.

11:40–12:30 **V. Bergelson** – *Ergodic theory along polynomials and combinatorial number theory*

**Abstract:** Various results obtained in recent years indicate that dynamical systems exhibit regular behavior along polynomial times. While being of interest in their own right, the polynomial recurrence and convergence theorems also lead to strong applications in combinatorics and number theory. We will discuss some of these applications including various polynomial extensions of Szemerédi's theorem on arithmetic progressions and the recent work of Tao and Ziegler on polynomial patterns in primes. We shall conclude by formulating and discussing some open problems and conjectures.

**Afternoon session chairperson: M. Rudelson**

14:00–14:50 **V. Pestov** – *Ramsey–Dvoretzky–Milman phenomenon: A survey*

**Abstract:** Vitali Milman had initiated the systematic study of a relationship between the concentration of measure and Ramsey-type statements in the context of groups of automorphisms of various mathematical objects as expressed in his dictum that "small local oscillations imply small global oscillations on smaller substructures". We survey a number of developments in this direction over the past four decades.

15:00–15:50 **J.M. Bismut** – *The hypoelliptic Laplacian and its applications*

NEW TIME

16:20–18:40 Mini-symposium in *Probability in Convex Bodies*

Organizers: B. Klartag and R. Vershynin

Participants:

**R. Latała** – *Problems related to concentration inequalities for logarithmically concave measures*

**E. Milman** – *Isoperimetry and concentration are equivalent under curvature lower bound*

**K. Oleszkiewicz** – *Vector-valued Rademacher sums*

**G. Paouris**

**M. Shcherbina** – *Central limit theorem for fluctuations of eigenvalues of large random graphs*

**Abstract:** We discuss the behavior of fluctuations of linear eigenvalue statistics of adjacency matrices of large random graphs  $\Gamma = \Gamma_{N,p}$  under assumptions that the edges of  $\Gamma = \Gamma_{N,p}$  are drawn independently and the average number of edges attached to one vertex  $p$  does not depend on  $N$ . It is shown that for any analytic test function the fluctuation converges in the distribution to the normal random variable.

**S. Sodin** – *Band random matrices: The edge of the spectrum*

**Abstract:** We shall try to explain what is a band matrix, how its extremal eigenvalues are distributed, and how this is related to the Thouless criterion and to the work of Fyodorov and Mirin.

**A. Volčič** – *Random Steiner symmetrizations of convex bodies and of measurable sets*

NEW TIME

18:45 **Supper**

20:00 *Concert*

20 minutes after concert: Buses leave Tel Aviv University for hotels, main gate

**Tuesday, June 30**

**Morning session chairperson: Y. Gordon**

9:00–9:50 **G. Kalai** – *Noisy quantum computers above the "Threshold"*

**Abstract:** The notion of quantum computers and quantum computation have led to exciting developments in theoretical computer science and various areas of physics. It also led to many interesting mathematical problems. The feasibility of computationally superior quantum computers is one of the most fascinating scientific problems of our time. After giving a brief introduction to the subject I will describe models of noisy quantum evolutions with unsuppressed noise-propagation and related adversarial models of noise. Such models may be of interest both in the context of natural open quantum systems and in trying to explore fundamental obstructions for quantum computers.

10:10–11:00 **Y. Sinai** – *Non-standard ergodic theorems and limit theorems*

**Abstract:** In this talk I shall review new ergodic theorems and limit theorems which appeared recently in problems related to ergodic theory and number theory.

NEW TITLE

11:10–12:00 **S. Novikov** – *New Discretization of Complex Analysis*

**Abstract:** Classical Discretization of the Cauchy-Riemann operator is associated with quadrilateral lattice. Few years ago (in 2002) we observed that Equilateral Triangle Lattice leads to much better discretization. I. Dynnikov and myself started to develop this subject. Other people also joined us later. Our ideas are borrowed from the theory of completely integrable "KdV type" systems.

**Afternoon session chairperson: A. Koldobsky**

13:30–14:20 **M. Rudelson** – *Invertibility and spectrum of random matrices*

**Abstract:** We will discuss several recent developments related to invertibility and spectral properties of large random matrices. Consider an  $n \times n$  matrix, whose values are independent identically distributed random variables. The invertibility questions can be roughly divided in two categories.

- Qualitative problems.

- What is the probability that a random matrix is invertible?

This probability is obviously 1, if the entries are absolutely continuous. However, in the case of discrete entries the problem becomes highly non-trivial.

- Quantitative problems.

- What is the typical distance between a random matrix and the set of singular matrices?

- How is this distance distributed?

We will briefly survey recent results concerning the qualitative questions. Most of the talk will be devoted to quantitative results, which are usually more involved. In particular, we will describe recent solutions of the von Neumann and Spielman–Teng conjectures about the behavior of the least singular value. Applications of these results to classical random matrix theory will also be considered.

14:30–15:20 **R. Schneider** – *Classical convexity and stochastic geometry*

**Abstract:** There have always been close relations between convex geometry and geometric probabilities. The more obvious of these, like the much studied topic of convex hulls of random points, will be excluded here (although there have been important developments in the last years). Rather, the talk tries to give a picture, including recent new results, of the role that valuations and auxiliary convex bodies play in the investigation of two prominent models of stochastic geometry, namely homogeneous Poisson processes of particles or of hyperplanes. Some celebrated classical results from convex geometry have interpretations and applications in this area. For example, Minkowski’s existence theorem is needed to investigate shapes of typical faces in random mosaics generated by hyperplane processes.

15:50 – **Short seminars:** Impromptu sessions initiated by participants.

**H. Rosenthal** – *The closeability for algebras of operators on a complex Banach space*

**Abstract:** Let  $A$  be a strongly closed algebra of (bounded linear) operators on a complex Banach space  $X$ .  $A$  is said to be the closeability property (cp) if every densely defined operator which commutes with  $A$  is closeable.  $A$  is said to have the strong closeability property (scp) if every densely defined operator which commutes with  $A$  has a unique closed extension which commutes with  $A$ . Also, an operator on  $X$  is said to have the cp if the strongly closed algebra it generates has it.

CONJECTURE: *If  $A$  has the cp, either  $A$  is all of  $L(X)$  or  $A$  has a (nontrivial closed linear) invariant subspace.* Deep work of Arveson from the 60s shows that MASAs in  $B(H)$  have the cp, and also our conjecture holds for any (strongly closed) algebra containing such. We also established the same for the unilateral shift. However, recent joint work of Bercovici, Douglas and Foias yields that the bilateral shift fails the cp.

We also discuss remarkable results due to U. Haagerup and P. Ainsworth (1985), which show that if  $N$  is a von Neumann algebra with finite commutant, then  $N$  has the scp, and investigate the Conjecture in the special case where  $A$  contains such an  $N$ . The cp and results discussed in joint work with Timur Oikhberg and Vladimir Troitsky.

16:00 One bus leaves TAU for hotels. The second bus will leave at a time to be announced, depending on the seminar’s program. Both leave from gate 2.