Name and ID of the student:

03.07.2015, moed A

Tel-Aviv University

Engineering Faculty

Final exam on "Differential and Integral Calculus"

Lecturer: Prof. Yakov Yakubov

Prescriptions:

1. The duration of the exam is 3 hours.
2. The use of any material is forbidden except the plane calculator and three personal lists (6 pages) of formulas, **including a list of quadratic surfaces**, prepared by the student. The size of the lists is the standard A4 format.
3. Do not use any methods which have not been studied in the classes.

The structure of the final exam:

1. There are 5 questions in the exam. You should answer to **only** 4 questions.
2. The grade of each question is 25 points.
3. Indicate on the first page of the exam which questions should be checked.
4. In the case you solve all 5 questions and you do not indicate which questions should be checked, first 4 questions will be checked.

Good luck!

Question 1

(a) (10 points) Calculate the limit $\lim_{n\to\infty}\sqrt[n]{3n-\sqrt{n}}$ **.**

(b) (15 points) Find the radius and interval of convergence (including endpoints) of the power series $\sum_{n=1}^{\infty} a^{n^2}(x-1)^n$. Hint: one should consider three cases of the constant $a>0$.

Question 2 Given a series $\sum_{n=1}^{\infty}\frac{(-1)^n}{x^4+\sqrt{n}}$.

(a) (4 points) For which values of x does the series converge?

(b) (6 points) For which values of x does the series absolutely converge?

(c) (15 points) For which largest domain of x does the series uniformly converge?

Question 3

(a) (13 points) Calculate the double integral $\iint_D \sin(xy)\,dA$, where D is a domain which is in $y\geq 0, x\geq 0$ and bounded by $xy=2$, $xy=4$, $y=x$ and $y=2x$.

(b) (12 points) Find the limit $\lim_{(x,y)\to(1,0)}(x-1+y)\sin\left(\frac{1}{(x-1)^2+y^2}\right)$ and prove your answer or prove that the limit does not exist.

Question 4

(25 points) Verify the Gauss theorem for the vector-field $\vec{F}=(x,y,xz)$ and S is a lateral surface of a body which is bounded by the cone $z=\frac{1}{2}\sqrt{x^2+y^2}$ and the plane $z=1$. The inward normal on S is given.

Question 5

(a) (13 points) Given the numerical sequence $a_1=5$, $a_{n+1}=\sqrt{3+2a_n}$. Prove that the sequence converges and find the limit of the sequence.

(b) (12 points) Check absolute convergence/ conditional convergence/ divergence of the series $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}}\ln(1+\frac{1}{n})$.