Name and ID of the student:

16.07.2015, moed B

Tel-Aviv University

Engineering Faculty

Final exam on "Differential and Integral Calculus"

Lecturer: Prof. Yakov Yakubov

Prescriptions:

1. The duration of the exam is 3 hours.
2. The use of any material is forbidden except the plane calculator and three personal lists (6 pages) of formulas, **including a list of quadratic surfaces**, prepared by the student. The size of the lists is the standard A4 format.
3. Do not use any methods which have not been studied in the classes.

The structure of the final exam:

1. There are 5 questions in the exam. You should answer to **only** 4 questions.
2. The grade of each question is 25 points.
3. Indicate on the first page of the exam which questions should be checked.
4. In the case you solve all 5 questions and you do not indicate which questions should be checked, first 4 questions will be checked.

Good luck!

Question 1

(a) (10 points) Calculate the limit $\lim_{n\to\infty}\sqrt[n]{1^4+2^4+\ldots+n^4}$ **.**

(b) (15 points) Find the radius and interval of convergence (including endpoints) of the power series $\sum_{n=1}^{\infty}(-1)^n nx^{n-1}$ and calculate the sum.

Question 2

(a) (12 points) Given a sequence of the functions $f_n(x)=x^2e^{-nx}$, where $n=1,2,3,...$

Does the series of functions $\sum_{n=1}^{\infty} f_n(x)$ converge uniformly on $[0,\infty)$? Prove.

(b) (13 points) Calculate the surface area of a part of the sphere $x^2+y^2+z^2=1$ which is situated between the planes $z=0$ and $z=\frac{1}{2}$.

Question 3

(25 points) Calculate the triple integral $\iiint_E z^3 e^{\left(x^2+y^2+z^2\right)^{3/2}}\,dV$, where E is a body which is in $z\ge 0, y\ge 0, x\ge 0$ and bounded by $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$.

Question 4

(25 points) Verify the Stokes theorem for the vector-field $\vec{F}=(yz,1,z)$ and C is an intersection curve between the cylinder $x^2+y^2=4$ and the half-sphere $x^2+y^2+z^2=16, z\ge 0$. The orientation on C is clockwise by observing from above.
Remark. For surface S one can choose any smooth surface with boundary C.

Question 5

(a) (13 points) Given the numerical sequence $a_1=c$, $a_{n+1}=\frac{a_n^2+c^2-1}{2c}$, where $c>1$.

Prove that the sequence converges and find the limit of the series.

(b) (12 points) Check absolute convergence/ conditional convergence/ divergence of the series $\sum_{n=1}^{\infty}(-1)^n\cos\left(\frac{1}{n^2}\right)$ and $\sum_{n=1}^{\infty}\left(\sqrt[n]{n}-1\right)^n$.