ID of the student:

24.01.2016, moed A

Tel-Aviv University

Engineering Faculty

Final exam on "Calculus 1B"

Lecturer: Prof. Yakov Yakubov

Prescriptions:

1. The duration of the exam is 3 hours.
2. The use of any material is forbidden except the plane calculator and two personal lists (4 pages) of formulas prepared by the student. The size of the lists is the standard A4 format.
3. Do not use any methods which have not been studied in the classes.

The structure of the final exam:

1. There are 5 questions in the exam. You should answer to **only** 4 questions.
2. The grade of each question is 25 points.
3. Indicate on the first page of the exam which questions should be checked.
4. In the case you solve all 5 questions and you do not indicate which questions should be checked, first 4 questions will be checked.

Good luck!

Question 1 (25 points)

Investigate and draw a graph of the function $y = f(x) = x\ln|x|$ (the domain of definition, the intersection points with the coordinate axis, symmetry, extreme points, monotonicity, convexity, inflection points, asymptotes, the graph).

Question 2

(a) (14 points) Given the sequence $a_n = \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}$. Prove that the sequence converges. Hint: show that the sequence is decreasing and bounded from below.

(b) (11 points) Prove that, for $0 < x < 1$, it holds $x < \arcsin x < \frac{x}{\sqrt{1-x^2}}$.

Question 3

(a) (11 points) Check the series $\sum_{n=1}^{\infty} \frac{n^2 \cos n}{n^{3.5}+1}$ for convergence/divergence.

(b) (14 points) Calculate the indefinite integral $\int \frac{x^3+x}{x^2+x+1}dx$.

Question 4

(a) (13 points) Check the integral $\int_{e^2}^{+\infty} \frac{1}{x\ln(\ln x)}dx$ for convergence/divergence.

Remark: if necessary, use that $s \ge \ln s > 0, \quad \forall s > 1$.

(b) (12 points) How many solutions there are for the equation $\ln x - x + 2 = 0$ on the interval $(0,+\infty)$? Justify the answer.

Question 5

(a) (12 points) Show, by the Weierstrass M-test, or by any other method, that the series of functions $\sum_{n=1}^{\infty} x^2 e^{-nx}$ uniformly converges on $[0,\infty)$.

(b) (13 points) Given a continuous function $g(x)$ such that $g(a) \ne 0$. Prove, by the definition of the derivative, that the function $f(x) = |x-a|\,g(x)$ is not differentiable at a.