ID of the student:

08.02.2017, moed A

Tel-Aviv University
Engineering Faculty

Final exam on "Calculus 1B"

Lecturer: Prof. Yakov Yakubov

Prescriptions:

1. The duration of the exam is 3 hours.
2. The use of any material is forbidden except the plane calculator and two personal lists (4 pages) of formulas prepared by the student. The size of the lists is the standard A4 format.
3. Do not use any methods which have not been studied in the classes.

The structure of the final exam:

1. There are 5 questions in the exam. You should answer to **only** 4 questions.
2. The grade of each question is 25 points.
3. Indicate on the first page of the exam which questions should be checked.
4. In the case you solve all 5 questions and you do not indicate which questions should be checked, first 4 questions will be checked.

Good luck!

Question 1 (25 points)

Investigate and draw a graph of the function $y = f(x) = \dfrac{\ln x}{\sqrt{x}}$ (the domain of

definition, the intersection points with the coordinate axis, symmetry, extreme points, monotonicity, convexity, inflection points, asymptotes, the graph).

Question 2

(a) (12 points) Calculate the limit

$$\lim_{n\to\infty} \frac{1}{n}\left(\frac{1}{\sqrt[n]{n^2+1}} + \frac{1}{\sqrt[n]{n^2+2}} + \cdots + \frac{1}{\sqrt[n]{n^2+n}} \right).$$

Hint: one can use the sandwich rule and $\lim\limits_{n\to\infty} \sqrt[n]{n} = 1$.

(b) (13 points) Prove that, for $0 < y < x$, it holds $\dfrac{1}{2x^2} < \dfrac{\ln x - \ln y}{x^2 - y^2} < \dfrac{1}{2y^2}$.

Question 3

(a) (10 points) Find the limit $\lim\limits_{x\to 0}(x + e^{2x})^{\frac{2}{x}}$.

(b) (15 points) Calculate the indefinite integral $\int \ln(x^3 + 3x)dx$.

Hint: integrate by parts and get an integral of a rational function.

Question 4

(a) (13 points) Check the integral $\int\limits_{1}^{+\infty} \dfrac{1}{e^x - 1}dx$ for convergence/divergence.

(b) (12 points) Check the series $\sum\limits_{n=1}^{\infty} \dfrac{n^{n+1}}{2^n n!}$ for convergence/divergence.

Question 5

(a) (13 points) Prove, by the corresponding Taylor's formula for $n = 2$, or by any

other method, that for any $x > 0$ it is true $\sqrt{1 + 2x} > 1 + x - \dfrac{1}{2}x^2$.

(b) (12 points) Calculate $\int\limits_{0}^{\pi} \sum\limits_{n=1}^{\infty} \dfrac{\cos(nx)}{n^{1.5}}dx$ if it exists.

Hint: Prove that the series converges uniformly on $[0, \pi]$ and change the order

between integration and infinite summation.