ID of the student:

21.02.20, moed A

Tel-Aviv University

Engineering Faculty

Final exam on "Calculus 1B"

Lecturer: Prof. Yakov Yakubov

Prescriptions:

1. The duration of the exam is 3 hours. Write the ID number on the top.
2. The use of any material is forbidden except the attached list of formulas.
3. Do not use any methods which have not been studied in the classes.
4. It is forbidden to keep any electronic device close to the exam place.
5. There are 4 questions in the exam. You should answer the all questions.
6. The grade of each question is 27 points but the maximal grade on the exam does not exceed 100.

Good luck!

Question 1

(a) (13 points) Calculate the integrals: $\int \frac{x^3}{x^2+4x+5}dx$, $\int \frac{x}{cos^2x}dx$ **.**

(b) (14 points) Given the equation $arctg(x) + x = 1$. Prove that there is a solution to the equation and the solution is unique.

Question 2

(a) (13 points) Prove that $\forall a \in \mathbb{R}$ it is true that $\lim_{n\to\infty} \frac{[a \cdot n]}{n} = a$, where $[x]$ denotes the lower integer part of x.

(b) (14 points) Calculate, by definition, the derivative of the function $f(x) = \ln(x^2+3)$ at any real point x_0. Remark: the use of the L'Hospital rule at any step of the solution is forbidden.

Question 3

(a) (10 points) Given a continuous function $f(x)$ on [0,2] such that $f(0) = f(2)$. Prove that there exists a point $x_0 \in (0,2)$ such that $f(x_0) = f(x_0+1)$.

(b) (10 points) Calculate the all possible asymptotes of the function

$f(x) = |x| + arctg\left(\frac{1}{x}\right)$

(c) (7 points) Calculate the limit $\lim_{x\to 0}(\cos(x) + x^2)^{1/x^2}$.

Question 4

(a) (9 points) Find all local extrema points of the function $f(x) = x^{4/3} - 2x^{2/3}$**.**

(b) (10 points) Prove, using the Lagrange mean-value theorem, that $\tan(x) > x$ for any $0 < x < \frac{\pi}{2}$.

(c) (8 points) Does the series $\sum_{n=1}^{\infty}(-1)^n \sin\left(\frac{\sin(\frac{1}{n})}{n}\right)$ converge absolutely, converge conditionally, or diverge?