ID of the student:

10.09.20, moed B

Tel-Aviv University

Engineering Faculty

Final exam on "Calculus 1B"

Lecturer: Prof. Yakov Yakubov

Prescriptions:

1. The duration of the exam is 3 hours. Write the ID number on the top.
2. The use of any material is forbidden except the attached list of formulas.
3. Do not use any methods which have not been studied in the classes.
4. It is forbidden to keep any electronic device close to the exam place.
5. There are 4 questions in the exam. You should answer the all questions.
6. The grade of each question is 27 points but the maximal grade on the exam does not exceed 100.

Good luck!

Question 1

(a) (14 points) Calculate the integrals: $\int \frac{x^3+x}{x^2+4x+6}dx$, $\int \frac{x}{sin^2 x}dx$.

(b) (13 points) Given the equation $e^x + x = 0$. Prove that there is a solution to the equation and the solution is unique.

Question 2

(a) (13 points) Prove, using the Cauchy definition of the limit, that

$$\lim_{x\to 0^+} \frac{\sqrt{x}+2x+2}{x+1} = 2$$

(b) (14 points) Calculate, by the derivative definition, the derivative of the function $f(x) = \cos(x^2)$ at any real point x_0. Remark: the use of the L'Hospital rule at any step of the solution is forbidden.

Question 3

(a) (14 points) Given a continuous function $f(x)$ on $[a,b]$ such that $f(a) = f(b)$. Prove that there exists a point $x_0 \in \left[a, \frac{a+b}{2}\right]$ such that $f(x_0) = f(x_0 + \frac{b-a}{2})$.

(b) (13 points) Calculate increasing and decreasing regions, local extrema points (minimum and maximum), and all possible asymptotes of the function $f(x) = \frac{|x-4|}{\sqrt{1+x^2}}$.

Question 4

(a) (9 points) Does the series $\sum_{n=1}^{\infty} \frac{(1+2n+3n^2)\arctan(n^2)}{\sqrt{3n^2+n^7}}$ converge? Prove.

(b) (9 points) Given the sequence by $a_1 = -1$ and $a_{n+1} = -a_n\sqrt{2+(-1)^{n+1}a_n}$ for any $n \in \mathbb{N}$. Does the sequence converge? If yes, calculate the limit, if not – prove why not.

(c) (9 points) Given a differentiable function $f(x)$ on $[2,b]$ such that $f(2) = 3$. Given also that $1 \le f'(x) \le 5$, $\forall x \in [2,b]$. Prove that it is satisfied, on the interval, $1 + x \le f(x) \le 5x - 7$.