ID of the student:

23.06.2016, moed A

Tel-Aviv University

Engineering Faculty

Final exam on "Calculus 2B"

Lecturer: Prof. Yakov Yakubov

Prescriptions:

1. The duration of the exam is 3 hours.
2. The use of any material is forbidden except the plane calculator and three personal lists (6 pages) of formulas, **including a list of quadratic surfaces**, prepared by the student. The size of the lists is the standard A4 format.
3. Do not use any methods which have not been studied in the classes.

The structure of the final exam:

1. There are 5 questions in the exam. You should answer to **only** 4 questions.
2. The grade of each question is 25 points.
3. Indicate on the first page of the exam which questions should be checked.
4. In the case you solve all 5 questions and you do not indicate which questions should be checked, first 4 questions will be checked.

Good luck!

Question 1 **(a) (12 points)** Given the function $f(x,y)=\begin{cases}\dfrac{x^4+y^4}{x^2+y^2}, & (x,y)\neq(0,0),\\ 0, & (x,y)=(0,0)\end{cases}$.

Calculate $f_x(x,y)$ and $f_y(x,y)$ for any $(x,y)\in\mathbb{R}^2$.

(b) (13 points) Check if the function differentiable or not at any point $(x,y)\in\mathbb{R}^2$.

Question 2 **(a) (10 points)** Given a single variable function $f(t)$ twice differentiable.

Show that the function $u(x,y)=f(e^x+\cos y)$ satisfies $(u_{xx}-u_x)\sin y=-e^x u_{xy}$.

(b) (15 points) Given the vector field $\vec{F}=(ye^x+xye^x+\frac{1}{1+x^2}, xe^x)$. Show that $\vec{F}$ is a conservative vector field, find $f(x,y)$ such that $\vec{F}=\nabla f$, and calculate $\int_C \vec{F}\cdot\hat{T}ds$ when C is a curve $y=\sin(\frac{\pi}{2}x)$, $x:0\to 1$.

Question 3 **(a) (14 points)** Find all critical points of the function

$f(x,y)=(x^2+y^2)e^{-x^2}$ and classify them (local min/max or saddle points).

(b) (11 points) Calculate the iterative integral $\int_0^{2\sqrt{\ln 3}}\int_{y/2}^{\sqrt{\ln 3}} e^{x^2}dxdy$.

Question 4 **(a) (15 points)** Find, using the Lagrange multipliers method, the maximum value of the function $f(x,y,z)=xyz$ when (x,y,z) is situated on the straight line $\begin{cases}x+y+z=40\\ x+y-z=0\end{cases}$. Is there the minimum value?

(b) (10 points) Calculate, using the triple integral, the volume of a solid, which is a part of the infinite cylindrical solid $1\le x^2+y^2\le 2$ bounded from below by $z=0$ and from above by $z=\sqrt{x^2+y^2}$.

Question 5 **(25 points)** Calculate the flux of $curl\vec{F}$ through the surface, given parametrically $\vec{r}(u,v)=(u\cos v, u\sin v, u)$, $0\le u\le 1, 0\le v\le 2\pi$, where $\vec{F}=(x^2y, 2y^3z, 3z)$. Is $\vec{F}$ a conservative vector field?