ID of the student:

10.07.2016, moed B

Tel-Aviv University

Engineering Faculty

Final exam on "Calculus 2B"

Lecturer: Prof. Yakov Yakubov

<u>Prescriptions:</u>

1. The duration of the exam is 3 hours.
2. The use of any material is forbidden except the plane calculator and three personal lists (6 pages) of formulas, **including a list of quadratic surfaces**, prepared by the student. The size of the lists is the standard A4 format.
3. Do not use any methods which have not been studied in the classes.

<u>The structure of the final exam:</u>

1. There are 5 questions in the exam. You should answer to <u>**only**</u> 4 questions.
2. The grade of each question is 25 points.
3. Indicate on the first page of the exam which questions should be checked.
4. In the case you solve all 5 questions and you do not indicate which questions should be checked, first 4 questions will be checked.

Good luck!

Question 1 (a) (13 points) Given the function

$$f(x,y)=\begin{cases}\dfrac{\ln(|x|+e^{|y|})}{\sqrt{x^2+y^2}}, & (x,y)\neq(0,0),\\ 1, & (x,y)=(0,0)\end{cases}$$

. Do the iterated limits $\lim_{x\to 0}(\lim_{y\to 0} f(x,y))$ and $\lim_{y\to 0}(\lim_{x\to 0} f(x,y))$ exist?

(b) (12 points) Does the limit $\lim_{(x,y)\to(0,0)} f(x,y)$ exist?

Question 2 (a) (15 points) Given a function of two variables $f(u,v)$ which satisfies the equation $f_{uu}+f_{vv}=0$. If $u=\dfrac{x^2-y^2}{2}$ and $v=xy$, show that the function $w=f(u,v)$ satisfies the equation $w_{xx}+w_{yy}=0$. Note that it is not given $f_{uv}=f_{vu}$.

(b) (10 points) Find the directional derivative of the function $h(x,y)=\sin x+y+1$ at the point $(0,1)$ in the direction $\vec{u}=(1,2)$. Calculate also $\max_{\hat{v}\in\mathbb{R}^2} D_{\hat{v}}h(0,1)$.

Question 3 (a) (14 points) Find all critical points of the function

$f(x,y)=\dfrac{1}{x^2+y^2-1}$, classify them (local min/max or saddle points), and find absolute min/max of the function in the disc $x^2+y^2\le \frac{1}{2}$.

(b) (11 points) Calculate the iterative integral $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{0}\frac{4\sqrt{x^2+y^2}}{1+x^2+y^2}\,dxdy$.

Question 4 (a) (15 points) Calculate the line integral

$\displaystyle\int_C (e^x\sin y-x)dx+(e^x\cos y-x^2+e^y)dy$ where C is the bottom half-circle of $(x+1)^2+y^2=1$ oriented clockwise.

(b) (10 points) Find all points on the cone $z=\sqrt{2x^2+2y^2}$ for which the tangent plane is parallel to the given plane $z=-x-y-9$.

Question 5 (a) (15 points) Calculate the flux of

$\vec{F}=(5x^3+12xy^2,\, y^3+e^y\sin z,\, 5z^3+e^y\cos z)$ through the surface S of a solid bounded by two spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=2$.

(b) (10 points) Is the above $\vec{F}$ a conservative vector field? What is the flux of $curl\vec{F}$ through any smooth closed surface S_0?