ID of the student:

12.07.2017, moed A

Tel-Aviv University

Engineering Faculty

Final exam on "Calculus 2B"

Lecturer: Prof. Yakov Yakubov

Prescriptions:

1. The duration of the exam is 3 hours.
2. The use of any material is forbidden except the plane calculator and three personal lists (6 pages) of formulas, **including a list of quadratic surfaces**, prepared by the student. The size of the lists is the standard A4 format.
3. Do not use any methods which have not been studied in the classes.

The structure of the final exam:

1. There are 5 questions in the exam. You should answer to **only** 4 questions.
2. The grade of each question is 25 points.
3. Indicate on the first page of the exam which questions should be checked.
4. In the case you solve all 5 questions and you do not indicate which questions should be checked, first 4 questions will be checked.

Good luck!

Question 1 **(a) (11 points)** Given the function $f(x,y)=\begin{cases} \dfrac{|x|^3+y^4}{x^2+y^2}, & (x,y)\neq(0,0),\\ 0, & (x,y)=(0,0)\end{cases}$

Do $f_x(0,0)$ and $f_y(0,0)$ exist? Is the function differentiable at $(0,0)$?

(b) (14 points) Calculate $f_x(x,y)$ and $f_y(x,y)$ at any point $(x,y)\neq(0,0)$. Is the function $f_x(x,y)$ continuous at $(0,0)$?

Question 2 **(a) (8 points)** Given two single variable differentiable functions $f(t)$ and $g(t)$. Show that the function $u(x,y)=f(e^{xy^2})+g(\sin(xy^2))$ satisfies $2xu_x-yu_y=0$.

(b) (17 points) Given $\vec{F}=(\frac{2xy^2}{1+x^2y^2}+y^2\cos(xy^2),\frac{2x^2y}{1+x^2y^2}+2xy\cos(xy^2)+1)$. Show that $\vec{F}$ is a conservative vector field, find $f(x,y)$ such that $\vec{F}=\nabla f$, and calculate $\int_C \vec{F}\cdot\hat{T}ds$ when C is the curve $\vec{r}(t)=(\sqrt{1-t^2},t),\ \ t:-1\to 1$.

Question 3 **(a) (13 points)** The function z of two variables x,y is an implicit function which is given by $xy+yz=\arctan(xz)$. Find formulas for $z_x(x,y)$ and $z_y(x,y)$. Calculate $z_x(1,0)$, $z_y(1,0)$ and the tangent plane to the graph of the function $z(x,y)$ at the point $(1,0)$.

(b) (12 points) Calculate the double integral $\iint_D \sin\left(\dfrac{\pi x}{2y}\right)dxdy$, where D is a domain defined by $y\geq x,\ \ y\geq\dfrac{1}{\sqrt{2}},\ \ y\leq\sqrt[3]{x}$.

Question 4 **(a) (13 points)** Find the absolute min/max for the function $f(x,y)=x^2-y^2+8$ in the domain $D=\{(x,y)\,|\,x^2+y^2\leq 4\}$.

Are there the absolute min/max for the function on the whole plane?

(b) (12 points) Calculate, using the triple integral, the volume of a solid, which is situated between the cone $z=\sqrt{2x^2+2y^2}$ and the half-sphere $z=\sqrt{3-x^2-y^2}$ above the upper-half disk: $x^2+y^2\leq 1, 0\leq y\leq 1$.

Question 5 **(25 points)** Justify the Gauss theorem for $\vec{F}=(x,y,z^2)$ and body E which is bounded by the cylinder $x^2+y^2=9$ and the planes $z=-1, z=3$.