ID of the student:

04.08.2017, moed B

Tel-Aviv University

Engineering Faculty

Final exam on "Calculus 2B"

Lecturer: Prof. Yakov Yakubov

Prescriptions:

1. The duration of the exam is 3 hours.
2. The use of any material is forbidden except the plane calculator and three personal lists (6 pages) of formulas, **including a list of quadratic surfaces**, prepared by the student. The size of the lists is the standard A4 format.
3. Do not use any methods which have not been studied in the classes.

The structure of the final exam:

1. There are 5 questions in the exam. You should answer to **only** 4 questions.
2. The grade of each question is 25 points.
3. Indicate on the first page of the exam which questions should be checked.
4. In the case you solve all 5 questions and you do not indicate which questions should be checked, first 4 questions will be checked.

Good luck!

Question 1 **(a) (13 points)** Prove, by the Cauchy definition, that the function

$$f(x,y)=\begin{cases}\dfrac{\ln(1+x^4+y^4)}{x^2+y^2}, & (x,y)\neq(0,0),\\ 0, & (x,y)=(0,0)\end{cases}$$ is continuous at $(0,0)$.

(b) (12 points) Find the directional derivative of the function at the point $(1,0)$ in the direction $\vec{u}=(-1,-1)$. Calculate also $\max_{\hat{v}\in\mathbb{R}^2} D_{\hat{v}} f(1,0)$.

Question 2 **(a) (13 points)** Given a function of a single variable $f(u)=u^{-\frac{1}{2}}$. If $u=x^2+y^2+z^2$, show that $f_{xx}+f_{yy}+f_{zz}=0$ at any point $(x,y,z)\neq(0,0,0)$.

(b) (12 points) Find all points on the ellipsoid $2x^2+2y^2+z^2=1$ for which the tangent plane is parallel to the given plane $z=-x+y-1$. Write the equation of the tangent plane at a point with positive x-coordinate.

Question 3 **(a) (14 points)** Find the absolute min/max for the function $f(x,y)=e^{-(x^2+y^2)}(2x^2+3y^2)$ in the domain $x^2+y^2\leq 4$. What are the points of the absolute min/max?

(b) (11 points) Calculate $\iint\limits_D xy\,dx\,dy$, when D is a domain (in $x>0, y>0$) bounded by circles $x^2+y^2=4, x^2+y^2=9$ and hyperbolas $x^2-y^2=1, x^2-y^2=3$.

Question 4 **(a) (15 points)** Calculate $\int\limits_C \frac{x+1}{\sqrt{(x+1)^2+y^2}}dx+\left(\frac{y}{\sqrt{(x+1)^2+y^2}}+\frac{x^2}{2}\right)dy$

where C is the left half-circle: $x^2+y^2=\frac{1}{4}, x\leq 0$, oriented clockwise.

(b) (10 points) Find all critical points of the function $f(x,y)=e^{x/2}(x+y^2)$ and classify them (local min/max or saddle points).

Question 5 **(a) (20 points)** Calculate the flux of

$\vec{F}=(x^3-\cos y, y^3+\sqrt{x^2+z^2}, z+5xy)$ through the positive oriented surface S which is a part of the elliptic paraboloid $z=4-x^2-y^2$ situated above the plane-xy.

(b) (5 points) Is the above $\vec{F}$ a conservative vector field?