ID of the student:

27.06.2018, moed A

Tel-Aviv University

Engineering Faculty

Final exam on "Calculus 2B"

Lecturer: Prof. Yakov Yakubov

Prescriptions:

1. The duration of the exam is 3 hours.
2. The use of any material is forbidden except the plane calculator and three personal lists (6 pages) of formulas, **including a list of quadratic surfaces**, prepared by the student. The size of the lists is the standard A4 format.
3. Do not use any methods which have not been studied in the classes.

The structure of the final exam:

1. There are 5 questions in the exam. You should answer to **only** 4 questions.
2. The grade of each question is 25 points.
3. Indicate on the first page of the exam which questions should be checked.
4. In the case you solve all 5 questions and you do not indicate which questions should be checked, first 4 questions will be checked.

Good luck!

<u>Question 1</u> **(a) (13 points)** Given the function $f(x,y)=\begin{cases}\dfrac{2xy}{x^2+y^2}, & (x,y)\neq(0,0),\\ 0, & (x,y)=(0,0)\end{cases}$

Calculate $f_{xy}(0,0)$ and $f_{xx}(0,0)$ if they exist.

(b) (12 points) Find all continuity points $(x,y)\in\mathbb{R}^2$ of the function. Is the function differentiable at $(0,0)$?

<u>Question 2</u> **(a) (10 points)** Given single variable functions $f(t)$ and $g(t)$ twice differentiable. Show that the function $u(x,y)=xf(x+y)+yg(x+y)$ satisfies $u_{xx}-2u_{xy}+u_{yy}=0$.

(b) (15 points) Given the vector field $\vec{F}(x,y,z)=(x,e^y\sin z,e^y\cos z)$. Show that $\vec{F}$ is a conservative vector field, find $f(x,y,z)$ such that $\vec{F}=\nabla f$, and calculate $\int_C \vec{F}\cdot\hat{T}ds$, where C is the curve $y=\sin(\frac{\pi}{2}x)$, $z=5$, $x:0\to 1$.

<u>Question 3</u> **(a) (14 points)** Find absolute minimum and maximum of the function $f(x,y)=x^2+9y^2$ in the domain D, where D is a closed triangle with vertices $(-1,1),(2,1),(-1,2)$.

(b) (11 points) Calculate the double integral $\iint_D \arctan\frac{y}{x}dA$, where

$D=\{(x,y):\ 1\le x^2+y^2\le 9,\ \ x/\sqrt{3}\le y\le x\sqrt{3}\}$. Hint: $\tan\pi/6=\frac{1}{\sqrt{3}},\tan\pi/3=\sqrt{3}$.

<u>Question 4</u> **(a) (15 points)** The curve C is an intersection line between the elliptic paraboloid $z=x^2+y^2$ and the plane $x+y+z=4$. Using the Lagrange multipliers, find on the curve the closest and the distant points from the point $(1,1,1)$. What are the corresponding minimal and maximal distances?

(b) (10 points) Calculate $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{2-x^2-y^2}(x^2+y^2)^{3/2}dzdydx$.

<u>Question 5</u> **(25 points)** Calculate $\int_C \vec{F}\cdot\hat{T}ds$, where $\vec{F}=(z^3,x^3,y^3)$ and C is an intersection line between the cylinder $x^2+y^2=2x$ and the plane $x+z=2$. The direction on C is counterclockwise by looking from above. Is $\vec{F}$ a conservative vector field? Calculate also $div(curl\vec{F})$. Is the answer by chance?