ID of the student:

15.07.2018, moed B

Tel-Aviv University

Engineering Faculty

Final exam on "Calculus 2B"

Lecturer: Prof. Yakov Yakubov

Prescriptions:

1. The duration of the exam is 3 hours.
2. The use of any material is forbidden except the plane calculator and three personal lists (6 pages) of formulas, **including a list of quadratic surfaces**, prepared by the student. The size of the lists is the standard A4 format.
3. Do not use any methods which have not been studied in the classes.

The structure of the final exam:

1. There are 5 questions in the exam. You should answer to **only** 4 questions.
2. The grade of each question is 25 points.
3. Indicate on the first page of the exam which questions should be checked.
4. In the case you solve all 5 questions and you do not indicate which questions should be checked, first 4 questions will be checked.

Good luck!

Question 1 **(a) (13 points)** Given the function

$$f(x,y)=\begin{cases}\dfrac{x+|y|}{\sqrt{x^2+y^2}}, & (x,y)\neq(0,0),\\ 1, & (x,y)=(0,0)\end{cases}$$

. Calculate $f_x(0,0)$, $f_y(0,0)$, and $f_x(1,0)$ if they exist.

(b) (12 points) Find all continuity points $(x,y)\in\mathbb{R}^2$ of the function. Is the function differentiable at $(0,0)$?

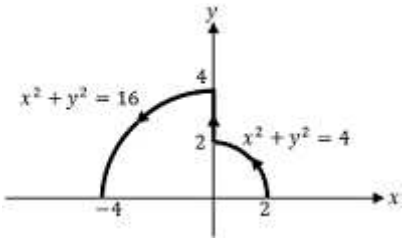
Question 2 **(a) (15 points)** Given a differentiable function of two variables $f(s,t)$ and $u(x,y,z)=x^2 f(s,t)$, where $s=\dfrac{y}{x}$ and $t=\dfrac{z}{x}$. Prove that $xu_x+yu_y+zu_z=2u$.

(b) (10 points) Find the directional derivative of the function $h(x,y)=\cos x+e^{xy}+1$ at the point $(0,1)$ in the direction $\vec{u}=(2,1)$. Calculate also $\max\limits_{\hat{v}\in\mathbb{R}^2} D_{\hat{v}}h(0,1)$.

Question 3 **(a) (12 points)** Find all critical points of the function $f(x,y)=x^3+y^3+3x^2-3y^2-8$ and classify them (local min/max or saddle points).

(b) (13 points) Calculate $\iint\limits_S z\sqrt{x^2+y^2}\,dS$, where S is parametrically given by $x=u\cos v,\ y=u\sin v,\ z=v$ and $0\le u\le 2, 0\le v\le 2\pi$.

Question 4 **(a) (15 points)** Calculate the line integral $\int\limits_C (-\frac{1}{2}y+2x)dx+(\frac{1}{2}x+y^2)dy$, where C is given on the illustration



(b) (10 points) Calculate the iterative integral $\int\limits_0^3\int\limits_0^{\sqrt{9-x^2}} \ln(1+x^2+y^2)\,dydx$.

Question 5 **(25 points)** Calculate the flux of $\vec{F}=(x^3-\cos y, y^3+\sqrt{x^2+z^2}, z+5)$ through the surface S which is a part of the elliptic paraboloid $x^2+y^2=4-z$ above the xy-plane. Is $\vec{F}$ a conservative vector field?