

Name and ID of the student:

06.02.2015, moed A

***Tel-Aviv University***  
**Engineering Faculty**

Final exam on "Differential and Integral Methods"

Lecturer: Prof. Yakov Yakubov

Prescriptions:

1. The duration of the exam is 3 hours.
2. The use of any material is forbidden except the plane calculator and three personal lists (6 pages) of formulas prepared by the student. The size of the lists is the standard A4 format.
3. Do not use any methods which have not been studied in the classes.

The structure of the final exam:

1. There are 5 questions in the exam. You should answer to **only** 4 questions.
2. The grade of each question is 25 points.
3. Indicate on the first page of the exam which questions should be checked.
4. In the case you solve all 5 questions and you do not indicate which questions should be checked, first 4 questions will be checked.

**Good luck!**

### **Question 1 (25 points)**

Investigate and draw a graph of the function  $y = f(x) = \frac{\ln x}{x}$  (the domain of definition, the intersection points with the coordinate axis, symmetry, extreme points, monotonicity, convexity, inflection points, asymptotes, the graph).

### **Question 2**

(a) (11 points) Calculate the limit  $\lim_{x \rightarrow +\infty} \left(1 + \frac{\ln x + 1}{e^x}\right)^{\frac{2e^x + 1}{\ln x + 2}}$ .

(b) (14 points) Given the function  $f(x) = \begin{cases} \frac{1}{|x|}, & |x| \geq 1, \\ ax^2 + b, & |x| < 1 \end{cases}$ . Find  $a$  and  $b$  such that

there exists  $f'(1)$ . Does  $f'(-1)$  exist for the same  $a$  and  $b$ ?

### **Question 3**

(a) (15 points) Calculate the improper integral  $\int_1^{+\infty} \frac{dx}{(x+1)^2(x+2)}$ .

(b) (10 points) Prove that for any  $0 < a < b$  and  $p > 1$  it is satisfied

$$(b-a)pa^{p-1} < b^p - a^p < (b-a)pb^{p-1}.$$

### **Question 4**

(a) (12 points) Find all critical points of the function  $f(x, y) = e^{x^2-y}(5+y)$  and classify them (local minimum and maximum points, saddle points).

(b) (13 points) Calculate  $\int_0^1 \left( \int_{\sqrt{x}}^1 \sqrt{x} \frac{\sin(\pi y^2)}{y^2} dy \right) dx$ .

### **Question 5**

(a) (10 points) Assume that  $u(x, y) = \sin x + f(\sin y - \sin x)$ , where  $f$  is a differentiable function of a single variable. Prove that

$$u_x \cos y + u_y \cos x = \cos x \cos y. \text{ Calculate } u_x(0,0) \text{ if } f'(0) = -1.$$

(b) (15 points) Calculate the line integral  $\int_C (2xye^{x^2y} + 3x^2y)dx + x^2e^{x^2y}dy$ , where

$C$  is the clockwise oriented ellipse  $x^2 + 4y^2 = 4$ .