Name and ID of the student:

26.02.2015, moed B

Tel-Aviv University

Engineering Faculty

Final exam on "Differential and Integral Methods"

Lecturer: Prof. Yakov Yakubov

Prescriptions:

1. The duration of the exam is 3 hours.
2. The use of any material is forbidden except the plane calculator and three personal lists (6 pages) of formulas prepared by the student. The size of the lists is the standard A4 format.
3. Do not use any methods which have not been studied in the classes.

The structure of the final exam:

1. There are 5 questions in the exam. You should answer to **only** 4 questions.
2. The grade of each question is 25 points.
3. Indicate on the first page of the exam which questions should be checked.
4. In the case you solve all 5 questions and you do not indicate which questions should be checked, first 4 questions will be checked.

Good luck!

Question 1 **(25 points)**

Investigate and draw a graph of the function $y = f(x) = \frac{x^2}{\sqrt{x^2 - 1}}$ (the domain of definition, the intersection points with the coordinate axis, symmetry, extreme points, monotonicity, convexity, inflection points, asymptotes, the graph).

Question 2

(a) (11 points) Calculate the limit $\lim\limits_{x \to 0} \frac{\sqrt{1 + \sin^2(2x)} - \sqrt{1 - \sin(2x)}}{x}$.

(b) (14 points) Assume that $f(x) = \begin{cases} 2(e^x - 1), & x \le 0, \\ \alpha e^{\sin x} + \beta \ln(1 + x), & x > 0 \end{cases}$. Find α and β such that there exists $f'(0)$.

Question 3

(a) (12 points) Calculate $\int \frac{x^4 + 1}{x^3 + x} dx$.

(b) (13 points) Write the Taylor's formula for the function $y = f(x) = \sqrt{x + 4}$ at $a = 0$ for $n = 2$, i.e., the Taylor's polynomial of order two and the Lagrange reminder for the third derivative. Using the formula, calculate approximately $\sqrt{5}$ and give an estimate of the “error”.

Question 4

(a) (13 points) Find minimum and maximum of the function $f(x, y) = \frac{1}{x} + \frac{1}{y}$ under the constraint $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{2}$. Specify the points (x, y) at which minimum and maximum are attained.

(b) (12 points) Find the volume of the solid which is bounded by the surfaces $z = x^2 + y^2$, $z = 2x^2 + 2y^2$, $y = x$, and $y = x^2$ (it is not necessary to give the exact picture of the solid).

Question 5

(a) (11 points) Find all points on the surface $xy^2 - z = 3$ at which the tangent plane to the surface is parallel to the given plane $4x + 8y - z = 7$.

(b) (14 points) Calculate the line integral $\int_C \left(x + \frac{1}{2}\sin(y^2) \right) dx + (xy\cos(y^2) + 4) dy$, where C is a straight line which starts at $(0, 0)$ and ends at $(-1, 3)$.