Name and ID of the student:

15.07.2015, moed B

Tel-Aviv University

Engineering Faculty

Final exam on "Partial Differential Equations"

Lecturer: Prof. Yakov Yakubov

Prescriptions:

1. The duration of the exam is 3 hours.
2. The use of any material is forbidden except the plane calculator and two personal lists (4 pages) of formulas prepared by the student. The size of the lists is the standard A4 format.
3. Do not use any methods which have not been studied in the classes.

The structure of the final exam:

1. There are 5 questions in the exam. You should answer to **only** 4 questions.
2. The grade of each question is 25 points.
3. Indicate on the first page of the exam which questions should be checked.
4. In the case you solve all 5 questions and you do not indicate which questions should be checked, first 4 questions will be checked.

Good luck!

Question 1 **(25 points)** Given the following wave problem

$$\begin{aligned} &u_{tt} - u_{xx} = 0, && 0 < x, \quad 0 < t, \\ &u(x,0) = x^3, \ \ u_t(x,0) = \sin x - x, && 0 \le x, \\ &u_x(0,t) = 0, && 0 \le t. \end{aligned}$$

(a) (20 points) Find the solution of the problem for any $x > 0, \ t > 0$.
(b) (5 points) Calculate $u(2,1)$ and $u(1,2)$.

Question 2 **(25 points)** Denote by $B = \left\{(x,y) : x^2 + y^2 < 4\right\}$. Solve the problem (here ∂B denotes the boundary of B):

$$\begin{aligned} &\Delta u = 0, \quad (x,y) \in B, \\ &\frac{\partial u(x,y)}{\partial n} = x^4 - y - 6, \quad (x,y) \in \partial B. \end{aligned}$$

Question 3 **(25 points)**

(a) (11 points) Classify the equation $u_{xx} + 6u_{xy} - 16u_{yy} = 0$, bring it to the canonical form and find its general solution.
(b) (14 points) Find the solution $u(r,\theta)$ of the boundary value problem:

$$\begin{aligned} &u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0, && 0 < r < 1, \ \ 0 < \theta < \tfrac{\pi}{4}, \\ &u(r,0) = u(r,\tfrac{\pi}{4}) = 0, && 0 \le r \le 1, \\ &u(1,\theta) = \sin(4\theta), && 0 \le \theta \le \tfrac{\pi}{4}. \end{aligned}$$

Question 4 (**25 points**)

(a) (18 points) Solve the Laplace equation in a rectangle:

$$\begin{aligned}
&\Delta u = 0, && 0 < x < \pi, \quad 0 < y < \pi, \\
&u(x,0) = 0, \quad u(x,\pi) = 0, && 0 \le x \le \pi, \\
&u(0,y) = 0, \quad u(\pi,y) = y(\pi - y), && 0 \le y \le \pi.
\end{aligned}$$

(b) (7 points) What are the minimum and the maximum of the solution in the closed rectangle?

Question 5 **(25 points)** Find the solution of the following non-homogeneous heat problem

$$\begin{aligned}
&u_t - 4u_{xx} = 1 + \tfrac{1}{2}\sin(2x), && 0 < x < \pi, \quad t > 0, \\
&u(0,t) = t, \quad u(\pi,t) = t + \pi, && t \ge 0, \\
&u(x,0) = \begin{cases} 2x, & 0 \le x \le \frac{\pi}{2}, \\ \pi, & \frac{\pi}{2} < x \le \pi. \end{cases}
\end{aligned}$$