Context Free Attentional Operators: the Generalized Symmetry Transform

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Abstract

Active vision systems, and especially foveated vision systems, depend on efficient attentional mechanisms. We propose that machine visual attention should consist of both high level, context dependent components, and low level, context free components. As a basis for the context free component, we present an attention operator based on the intuitive notion of symmetry, which generalizes many of the existing methods for detecting regions of interest. It is a low level operator that can be applied successfully without *a priori* knowledge of the world. The resulting *symmetry edge map* can be applied in various low, intermediate and high level tasks, such as extraction of interest points, grouping and object recognition. In particular, we have implemented an algorithm which locates interest points in real time, and can be incorporated in active and purposive vision systems. The results agree with some psychophysical findings concerning symmetry as well as evidence concerning selection of fixation points. We demonstrate the performance of the transform on natural, cluttered images.

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1 Introduction

Biological vision is foveated, highly goal oriented and task dependent. This observation, which is rather clear if we trace the behavior of practically every vertebrate, is now being taken seriously into consideration by the computer vision community. This is evident from recent work on active vision systems and heads [18, 11, 31] and general active vision concepts and algorithms [2, 6, 1, 7, 12]. One of the fundamental features of active vision is the use of space-variant vision and sensors [37, 34, 32], that allow, in the case of the log-polar representation data reduction as well as a certain degree of size and rotation invariance.

The use of such sensors require efficient mechanisms for gaze control, that are, in turn, directed by attentional algorithms. Using psychophysical terms, these algorithms are either *overt*, analyzing in detail the central foveated area, or *covert*, analyzing various regions within the field of view that are not necessarily in the central foveated area.

Like many other issues in computational vision, the attention problem seems to be trapped in the typical top down-bottom up cycle, as well as in the global-local cycle: global processes are necessarily based on local features and processes, whose crucial parameters, in turn, depend on global estimates. This is the case for recognition tasks, where, for example, thresholds and size tuning of local feature detectors are optimally determined by the model of the object the system expects. In curve and edge detection, local discontinuities are classified as signals or as noise according to global matching based on these very local estimates [40]. Similarly, attention is undoubtedly a concurrent top down *and* bottom up process: computational resources are assigned to regions of interest. But detection of regions of interest is both *context dependent* (top down), since the system is task oriented, and *context free* (bottom up), since one of the most important aspects of such a system is detection of unexpected signals. Thus, attention must be based on highly coupled low level and high level processes. While we do not offer a solution to this fundamental problem, we propose in this paper to begin this cycle with a low level attentional mechanism.

Visual processes, in general, and attentional mechanisms, in particular, seem effortless for humans. This introspection, however, is misleading. Psychophysical experiments show that infants (age 1-2 months) tend to fixate around an arbitrary single distinctive feature of the stimulus, like the corner of a triangle [17, 33]. Moreover, when presented with line drawings, children up to age 3-4 spend most of their time dwelling only on the internal details of a figure, and in general, children make more eye movements and are less likely than adults to look directly at a matching target in their first eye movements [10]. In comparison, adults display a strong tendency to look directly at forms that are informative, unusual, or of particular functional value [3, 23]. Thus, it seems that gaze control in adults is indeed task and context dependent, but it is probably based on natal (hardwired) low-level local and context free attentional mechanisms. At first, only the low level context free mechanisms are available. Gradually, as more information regarding the environment is being learned, higher level processes take their place.

Active vision definitely needs high level, context dependent attentional algorithms, but these should be adaptive, trainable algorithms based on acquired knowledge, that use lower level context free attentional modules. Considering the fact that this research area is rather new, we believe that robust and efficient low level attentional algorithms are the basic building blocks for machine visual attention. In this paper, we present an attentional algorithm, that is context free, does not depend on object segmentation, and seems to generalize many of the existing cues used in previous computer vision models for detecting regions of interest. While some of the motivation for our work is based on psychophysical observations, and our results are also in good agreement with psychophysical data relating overt attention, we do not present here our work as a model for overt and covert human attention.

2 Attentional mechanisms in Computer Vision

An early attentional operator based on grey level variance [26] is still being widely used in many systems. Other researchers suggested to measure *"busyness"* – the smoothed absolute value of the Laplacian of the data, rapid changes in the gray levels, and edge junctions [18]. Following early psychophysical findings [5, 20] interest points can be regarded also as points of high curvature of the edge map [21, 37].

Instead of selecting one of these methods for each application, we show that they can all be derived from a more generalized concept, and suggest that local generalized symmetry is the principle underlying context free attention. In addition, the operator we propose more closely fits psychophysical evidence, and it is more useful in detecting interesting features in complex scenes.

Natural and artificial objects often give rise to the human sensation of symmetry. Our sense of symmetry is so strong that most man-made objects are symmetric, and the Gestalt school considered symmetry as a fundamental principle of perception. Looking around us, we get the immediate impression that practically every interesting area consists of a qualitative and generalized form of symmetry.

In computer vision research symmetry has been suggested as one of the fundamental non-accidental properties, which should guide higher level processes. This sensation of symmetry is more general than the strict mathematical notion. For instance, a picture of a human face is considered highly symmetric by the layman, although there is no strict reflectional symmetry between both sides of the face.

Symmetry is being widely used in computer vision [13, 25, 16, 24, 4, 8, 15, 36, 39]. However, it is mainly used as a mean of convenient shape representation, characterization, shape simplification, or approximation of objects, which have been already segmented from the background. A schematic (and simplified) vision task consists of edge detection, followed by segmentation, followed by recognition. A symmetry transform is usually applied after the segmentation stage. We present a symmetry transform which is inspired by the intuitive notion of symmetry, and assigns a *symmetry magnitude* and a *symmetry orientation* to every pixel in an image at a low level vision stage which follows edge detection. Specifically, we compute a *symmetry map*, which is a new kind of an edge map, where the magnitude and orientation of an edge depends on the symmetry associated with the pixel. Strong symmetry edges are natural interest points, while linked lines are symmetry axes.

Since our symmetry transform can be applied immediately after the stage of edge detection, it can be used to direct higher level processes, such as segmentation and recognition, and can serve as a guide for locating objects. We shall now define the transform, and demonstrate these ideas on natural images.

3 Definition of the Transform

In the usual mathematical notion, an object is regarded symmetric if it is invariant to the application of certain transformations, called symmetry operations. A typical symmetry operation is the well known reflectional (mirror) symmetry. In order to use these symmetry operations it is necessary to know the shape of an object before we can estimate whether it is symmetric or not. However, the process of finding interest points must precede complex processes of detecting the objects in the scene. Even if the objects' shapes are known, truly symmetric objects are rare in natural scenes, and therefore any attempt to formulate an interest operator based on the strict mathematical notion of symmetry is doomed to fail.

Our symmetry transform does not require the knowledge of the object's shape. It performs local operations on the edges of the image. Moreover, it assigns a continuous symmetry measure to each point in the image, rather than a binary symmetry label.

We first define a symmetry measure for each point. Let $p_k = (x_k, y_k)$ be any point ($k = 1, \ldots, K$), and denote by $\nabla p_k = \left(\frac{\partial}{\partial_x} p_k, \frac{\partial}{\partial_y} p_k\right)$ the gradient of the intensity at point p_k. We assume that a vector $v_k = (r_k, \theta_k)$ is associated with each p_k such that $r_k = \log\left(1 + \|\nabla p_k\|\right)$ and $\theta_k = \arctan\left(\frac{\partial}{\partial_y} p_k / \frac{\partial}{\partial_x} p_k\right)$. For each two points p_i and p_j, we denote by l the line passing through them, and by α_{ij} the angle counterclockwise between l and the horizon. We define the set $\Gamma(p)$, a distance weight function $D_\sigma(i,j)$, and a phase weight function $P(i,j)$ as

$$\Gamma(p) \quad = \quad \left\{ (i,j) \,\middle|\, \frac{p_i + p_j}{2} = p \right\}$$

$$
\begin{aligned}
D_\sigma(i,j) &= \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\|p_i - p_j\|}{2\sigma}} \\
P(i,j) &= (1 - \cos(\theta_i + \theta_j - 2\alpha_{ij}))\,(1 - \cos(\theta_i - \theta_j))
\end{aligned}
$$

We define the contribution of the points p_i and p_j as

$$C(i,j) = D_\sigma(i,j) P(i,j) r_i r_j$$

This measure can be easily normalized, and reflects the fact that each of its components modulates the other ones. The *symmetry magnitude* or *isotropic symmetry* $M_\sigma(p)$ of each point p is defined as

$$M_\sigma(p) \quad = \sum_{(i,j)\in\Gamma(p)} C(i,j)$$

which averages the symmetry value over all orientations. We define the direction of the contribution of p_i and p_j as

$$\varphi(i,j) = \frac{\theta_i + \theta_j}{2}$$

A complementary definition for $\varphi(i,j)$ is given in section 6.

The *symmetry direction* is defined as $\phi(p) = \varphi(i,j)$ such that $C(i,j)$ is maximal for $(i,j) \in \Gamma(p)$. Thus, the symmetry of the point p is defined as

$$S_\sigma(p) = (M_\sigma(p), \phi(p))$$

The demand that the symmetry transform be local is reflected by the Gaussian distance weight function, $D_\sigma(i,j)$. Different values for σ imply different scales, thus enabling convenient implementation of multi-resolution schemes. Note that the Gaussian defined above has circular isotherms, i.e. it has no preferred orientation. However, one can also define Gaussians with elliptic isotherms. This is useful when the transform is applied as a feature detector of elliptic regions such as eyes in human faces [30, 14]. In the experimental results presented in this paper we have used only circular Gaussians.

The phase weight function, $P(i,j)$ is composed of two terms. The first term, $1 - \cos(\theta_i + \theta_j - 2\alpha_{ij})$, allows to achieve maximum symmetry when $(\theta_i - \alpha_{ij}) + (\theta_j - \alpha_{ij}) = \pi$ i.e. when the gradients at p_i and p_j are oriented in the same direction towards each other. This is consistent with the intuitive notion of symmetry. This expression decreases continuously as the situation deviates from the ideal one. Notice that the same measure is achieved when the gradients are oriented towards each other or against each other. The first situation corresponds to symmetry within a dark object on a light background, and the second corresponds to symmetry within a light object on a dark background. It is easy to distinguish between the two cases. (See Figure 2). One can simultaneously process both types of the transform obtaining two classes of interest points. However, for the simplicity of exposition, in all the experimental results presented in this paper, we consider only the case of gradients facing each other (which correspond mainly to dark objects on a brighter background). We emphasize that in general, both dark and bright object can be simultaneously detected. Obviously, the amount of false alarms will increase, but the basic idea behind any interest operator is dimensionality reduction: in the case of a 1000 by 1000 image, computational resources should be directed at, say, 10^2 locations, instead of 10^6. Thus, the increase in potential regions of interest caused by including bright and dark symmetry peaks is not significant in our context.

The second term of $P(i,j)$, $1-\cos(\theta_i-\theta_j)$, is introduced since the first term attains its maximum whenever $(\theta_i-\alpha_{ij})+(\theta_j-\alpha_{ij})=\pi$. This includes the case $\theta_i-\alpha_{ij}=\theta_j-\alpha_{ij}=\pi/2$, which occurs on a straight edge, which we do not regard as interesting. The current expression compensates for this situation.

The term $r_i r_j$ is high when there is a strong correlation between two large gradients. We use gradients rather than intensities since we are mainly interested in edges that relates to object borders. For instance, a uniform intensity wall is highly symmetric but probably not very interesting. In natural scenes we prefer to use the logarithm of magnitude instead of the magnitude itself, since it reduces the differences between high gradients, and therefore the correlation measure is less sensitive to very strong edges.

Note that the transform that we have defined detects reflectional symmetry. It is invariant under 2-D rotation and translation transformation. Moreover, it is quite effective in detecting skewed symmetry [19] as well. Since skewed symmetry results in an affine transformation of the 2-D picture, the midpoint of a segment and parallelism are conserved. Thus the location of the symmetry edge is preserved under skewed symmetry, but the direction of the symmetry edge should not be the average of the two directions. In section 6 we give the precise derivation of the correct direction. One should note, however, that since the weight of the direction prefers parallel edges, even the above defined direction will be still effective under skewed symmetry.

Sometimes it is necessary to detect points that are highly symmetric in multiple distinct orientations rather then in a principle one. We define such a symmetry as *radial symmetry* – $RS\,(p)$ and its value can be evaluated using the formula:

$$RS_\sigma\,(p) \quad = \sum_{(i,j)\in\Gamma(p)} C(i,j) \ \ \sin^2\left(\varphi(i,j)-\phi(p)\right)$$

This expression emphasizes contribution in the directions which are perpendicular to the main symmetry direction, and attains its maximum in a point that is surrounded by edges. Notice that due to the continuous nature of the operator, the radial symmetry is not sensitive to gaps in the contour that surrounds the point p, and does not require this contour to be uninterrupted. A preliminary definition of this symmetry form was suggested in [29].

Although the symmetry transform is presented here as a context-free mechanism, it can obviously use additional knowledge, if available. For example, if we look for a symmetry in a given direction, we define the *symmetry projection* at a point p and an orientation ψ as

$$PS_\sigma(p, \psi) = M_\sigma(p) \cos(\phi(p) - \psi)$$

The symmetry transform can thus be turned into an efficient feature detector. We have recently demonstrated this in [30], where the transform was used for detection of facial features.

The complexity of our algorithm is relatively low. We first discuss serial implementation. Suppose the operator is applied to a picture composed of n pixels, and the Gaussian in the weight function almost vanish in radius $r < \sqrt{n}$. Each two pixels, whose distance is less than or equal to $2r$, contribute a value which can be computed using look-up tables. Therefore the time complexity is $O(n\,r^2)$.

Space complexity is small too. The result of the operator are the symmetry magnitude and symmetry orientation maps, each occupies n pixels. In additional, a temporary space for holding the maximal response is needed. For the radial symmetry another n pixels map is needed.

The complexity of a parallel implementation depends on the architecture. For an architecture where there is a processor allocated for each pixel and each processor is connected to its neighbors up to radius r, we can achieve maximal speed-up of the algorithm and reduce time complexity to $O(r^2)$ (with $O(n\,r^2)$ messages if no global memory is available). If we have at our disposal a sufficiently large neural network, we can easily implement the look-up table and summation operations needed and then perform the operation in constant time. A different realization by "cortical-like" elements would be closely related to the co-circularity detectors suggested by [40].

4 Operation on natural images

The symmetry transform can be applied successfully on intricate natural scenes. It should be emphasized that all the images in figures 4-8 have been processed using the same parameters of the symmetry transform. Figure 3 demonstrates that the transform performance is not affected by the existence of several objects in the scene. One can also see that linking the strong symmetry edges results in natural symmetry axes of the parts involved. Notice that that the credit card is a skewed rectangle.

Figure 4 demonstrate the processing of an image of two persons on a noisy background. Note that the faces of the persons are marked among the highest symmetry peaks. Since the transform is designed to respond best to symmetric edges, this is not surprising. However, experimenting with many natural images, we have observed that, in spite of the context free, low level nature of the transform, it detects regions that humans would consider interesting.

Zooming in on figure 4 at the vicinity of the highest radial symmetry peak , we demonstrate (figure 5) the results of applying the transform with exactly the same parameters. The location of the strongest peaks of the radial symmetry transform are marked by crosses and are located on the eyes and mouth. This figure can also demonstrate the superiority of the symmetry transform over the direct search for dark blobs: such a search would mainly detect the hair, while we are intereseted in more intricated areas, which are designated by the local symmetric configuration of edges.

Notice that an active vision task typically involves exactly these steps: detecting an interest point, zooming in (which is equivalent to modifying the scale parameter), and using the same transform to detect regions of interest at the next level. This is also the stage where higher processes could use the low level modules for feature and object detection. In this regard, it might be useful for a higher level process to analyze in details the scale space patterns of the symmetry edge maps that are formed while zooming.

Figures 6 and 7 have been processed in the same manner as figures 4 and 5 with exactly the same parameters. Notice that this image includes many high frequency edges (e.g on the fur), yet the transform responds in a similar manner to figure 4, detecting the interesting objects on the coarse level, and the facial features on the fine level. This phenomenon should be further investigated, however, by analyzing the appropriate scale space.

Again, processing figure 8 with the same symmetry parameters as before, one can see the effectiveness of the symmetry transform in a different scene. Due to the continuous nature of the generalized symmetry transform, it gracefully degrades, and this is evident especially in cases of partial occlusion of symmetric objects, as can also be seen in this figure. Notice also that the same features are detected for the three foremost planes, though their size considerably vary.

Since symmetry is a powerful non-accidental property, significant information can be extracted from the image based on the symmetry map. In particular one can attempt direct segmentation of an image based on a backward transform which detects the image edges contributing to high symmetry. In figure 9 we show the result of ranking the edges according to their contribution to the symmetry maps. (See [30] for more details). The edges that caused high symmetry value can be used to segment the objects that were the source of this symmetry.

5 Conclusion

Stressing the role of low level context free attentional mechanisms, we have introduced a interest operator that associates a symmetry magnitude and direction with each pixel in an image. As opposed to previous paradigms the symmetry map is computed directly from the edge map of the image without prior segmentation.

The symmetry map can serve as an efficient low level process for indexing attention to the regions that are likely to be of high interest in a picture. Other processes can then direct computational resources to these regions and interpret the data in them. The symmetry transform agrees with some psychophysical data and can be further investigated to find its relation to models for human visual perception [9], and for indexing attention in biological vision, such as the models used by Posner and Peterson [27] and by Ullman [35].

There are many psychophysical works investigating humans ability to detect bilateral symmetry (e.g. [22]). It is accepted that humans are able to detect mirror symmetry and that symmetry detection has a local nature. However, there was almost no link suggested between these capabilities and the choice of attention points. Kaufman and Richards [20] studied spontaneous fixation tendencies of the eyes when they are confronted with very simple forms. Figure 10 shows the results for a collection of nine small figures. These results hold for forms which subtend less than $5°$ on the retina. As the forms become larger, fixation becomes scattered. Thus, it seems that the natural fixation mechanism has a local nature. These results are in intriguing agreement with the results of applying the symmetry transform to similar forms. As an example, consider the square in figure 11.

Another result , that might be more directly related to our operator, shows that symmetric patterns are more efficient in attracting attention [22]. It was found that when symmetric patterns were presented to subjects, both the survey and examination fixations were concentrated along the axes of symmetry. It is interesting to note that in this research, subjects were shown abstract art works in order to eliminate context.

Some psychophysical findings [5, 20] lead to the assumption that interest points can be regarded also as points of high curvature. However, using the symmetry transform at low resolution results in detection of points with high curvature (as shown in figure 11). This is partially due to the fact that the symmetry axis passes through the points with maximal curvature [38].

Our operator generalizes most of the existing methods for detection of interest points. The symmetry transform value is high near points of high curvature, since the edges that construct the high curvature are highly symmetric according to our definition. This is in agreement with the theoretical analysis in [38], which shows that axis of symmetry terminate at maximal curvature points. Corners also give rise to high symmetry values, since they consist of edges that point toward each other. Busyness measures can also be regarded as a rough estimate of the symmetry transform, since both are influenced by edge intensities. Note that busyness is insensitive to orientation, and thus will yield poor results in noisy scenes. Since most symmetry axes of an object pass through its center of gravity, the symmetry transform, and especially its radial form, highly responds to the center of gravity. However, as opposed to methods based on center of gravity, our operator does not require any segmentation or specification of the object's vertices.

There is, however, much more to attention than simple, context free mechanisms. As we have argued, attention, like almost every other visual task, is trapped in the local-global, top down - bottom up, and context free-context dependent vicious circle. A possible path to follow should consist of a three level paradigm:

- A context free direct computation of a set of simple and early mechanisms, like colour, motion or the generalized symmetry.
- An analysis of the geomtry of this early map, based on general, and task dependent knowledge.
- A conventional object detection and recognition [21] performed only in locations indexed by the previous stages.

This approach is not necessarily bottom up, as it seems to be on first sight, since the specific set of context free features used in the lower level could be selected and modified by the higher levels, once a specific task is performed. These modifications can include, for example, use of non symmetrical (oriented) Gaussians in the lower level, specification of the appropriate combination of dark and bright symmetries, and the use of the the symmetry projection. In the higher levels, the symmetry edge map obtained by the lower level can be operated on as an edge map applying standard computer vision tools. In particular, one can apply a Hough transform for line detection, thus detecting significant straight symmetry axes. By applying other standard edge linking procedure, one obtains all the symmetry axes in the image, which may be curved. Another analysis that can be useful at this stage is a detailed analysis of the scale space formed by the symmetry maps while zooming in.

The approach could be demonstrated, for example, in the task of detecting persons and facial features in images. We first compute the context free generalized symmetry map, and then we look for a geometric pattern where the symmetry peaks are vertically arranged, below a circular symmetry peak (persons), or three symmetry peaks that form a triangle (facial features). Indexed locations could then be specifically analyzed by edge based or intensity based recognition schemes.

Our main point in this work, was to propose a scheme for the first level of of the intricated process of attention, that generalizes most of the existing methods. The approach is stable, is proved to work on real, cluttered images, can tolerate partial occlusion of objects, can be implemented in real time, and thus is a promising method for early vision attentional mechanisms in purposive and active vision systems.

6 Appendix : Direction of the skewed symmetry edges

Consider an ideal object that possesses a reflectional symmetry. Let p_i and p_j be two contour points which are mirror symmetric with respect to the (symmetry axis) point p. Then, $p = \frac{p_i+p_j}{2}$. Let θ_i and θ_j be the orientation of the gradients at p_i and p_j respectively, then $\theta_i + \theta_j = \pi$, and the direction of the symmetry axis at p is $\frac{\theta_i+\theta_j}{2}$. The generalized symmetry transform presented here provides a continuous measure of symmetry and is designed also for cases where $\theta_1 + \theta_2 \neq \pi$. We have not assumed any cause for the deviation from the ideal situation, and therefore we have defined the symmetry orientation to be $\frac{\theta_i+\theta_j}{2}$ even in the non ideal case. Note that this definition is invariant under translation, rotation and change of scale (similarity transformation).

A perspective image of an ideal symmetric object is in general non symmetric. Thus, one would like to define a symmetry transform which is invariant to the perspective transformation. A common approximation to the perspective transformation is the affine transformation. This kind of symmetry is known as *skewed symmetry* [19].

Let us consider first the position of a skewed symmetry edge. Since a midpoint of a segment is an affine invariant, the position is the same as in the previous case. The definition of a skewed symmetry edge orientation is, however, different. It follows from basic trigonometric considerations [28] that the natural skewed symmetry edge orientation is

$$\varphi(i,j) = \arctan \frac{2\ \cos(\theta_i - \alpha_{i,j})\ \cos(\theta_j - \alpha_{i,j})}{sin(\theta_i + \theta_j - 2\alpha_{ij})}$$

where α_{ij} is defined as in section 3. One may need to add π to be consistent with the ideal case.

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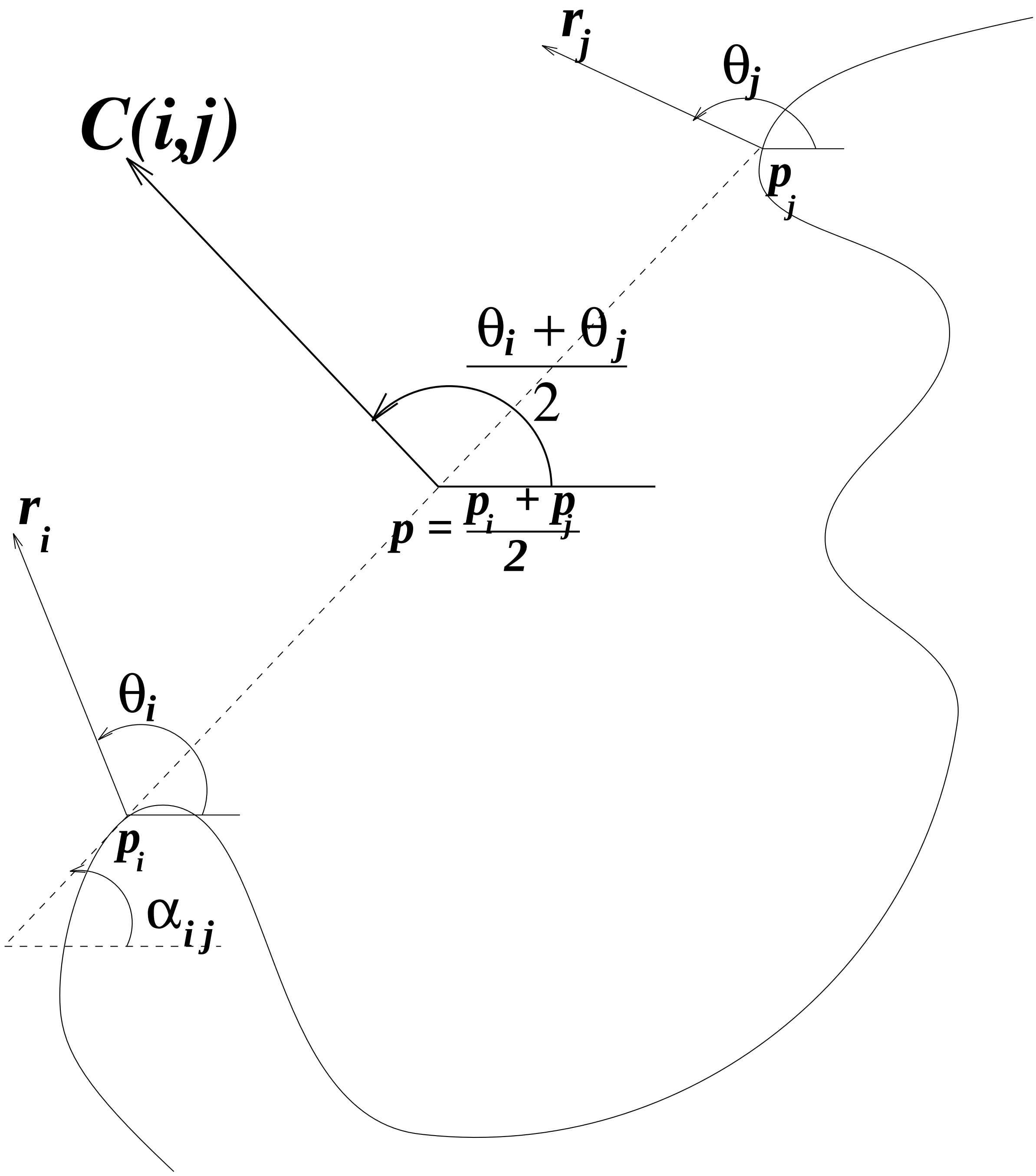
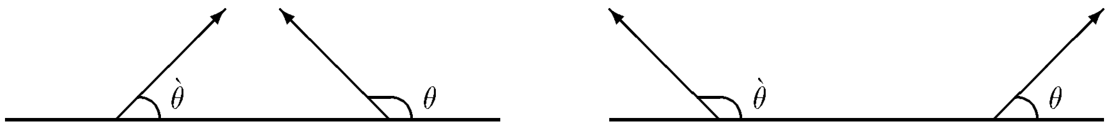
List of Figures



$\boldsymbol{r_j}$
θ_j
$\boldsymbol{C(i,j)}$
$\boldsymbol{p_j}$
$\dfrac{\theta_i + \theta_j}{2}$
$\boldsymbol{r_i}$
$\boldsymbol{p} = \dfrac{\boldsymbol{p_i} + \boldsymbol{p_j}}{2}$
θ_i
$\boldsymbol{p_i}$
α_{ij}

Figure 1:



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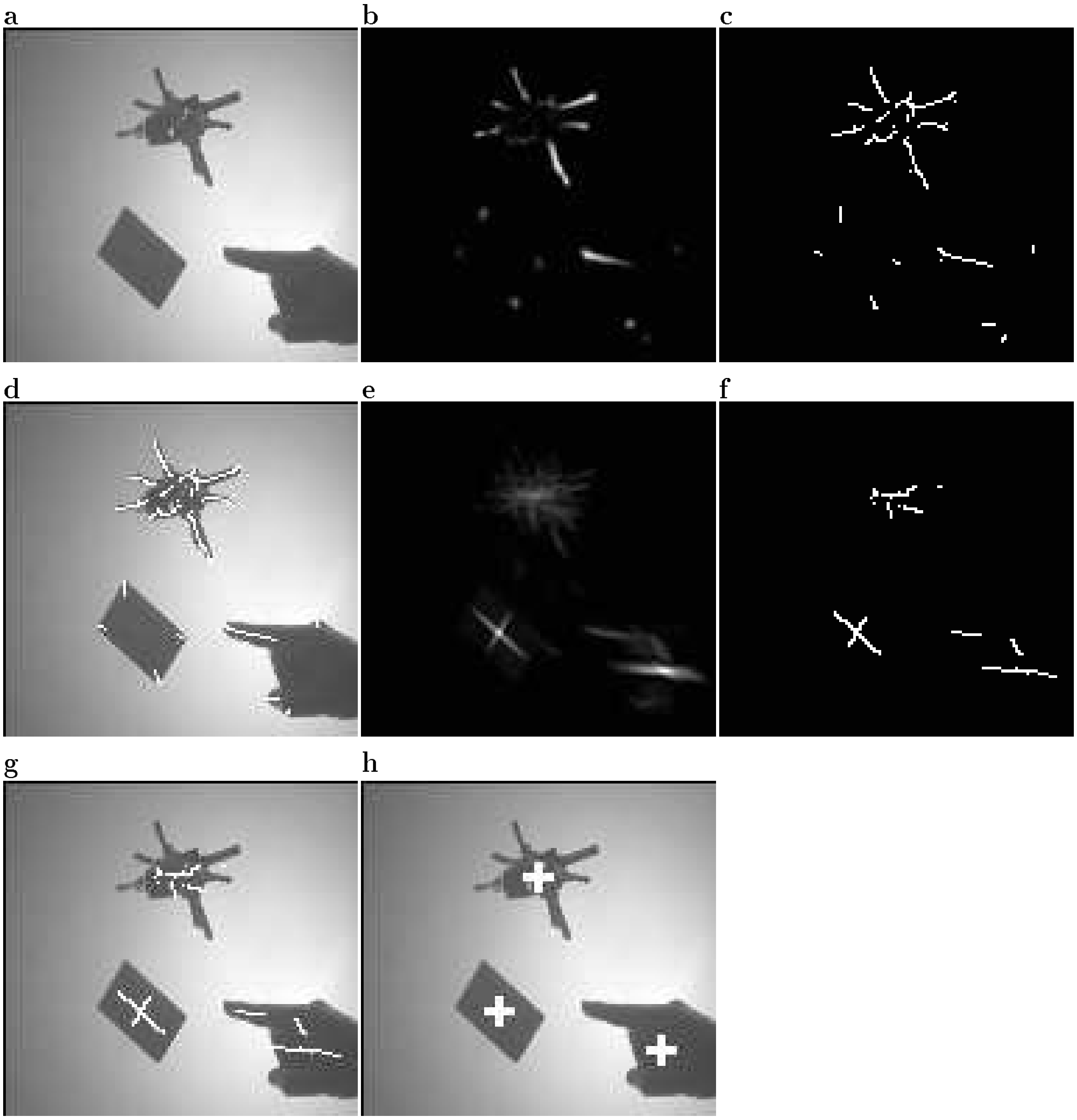

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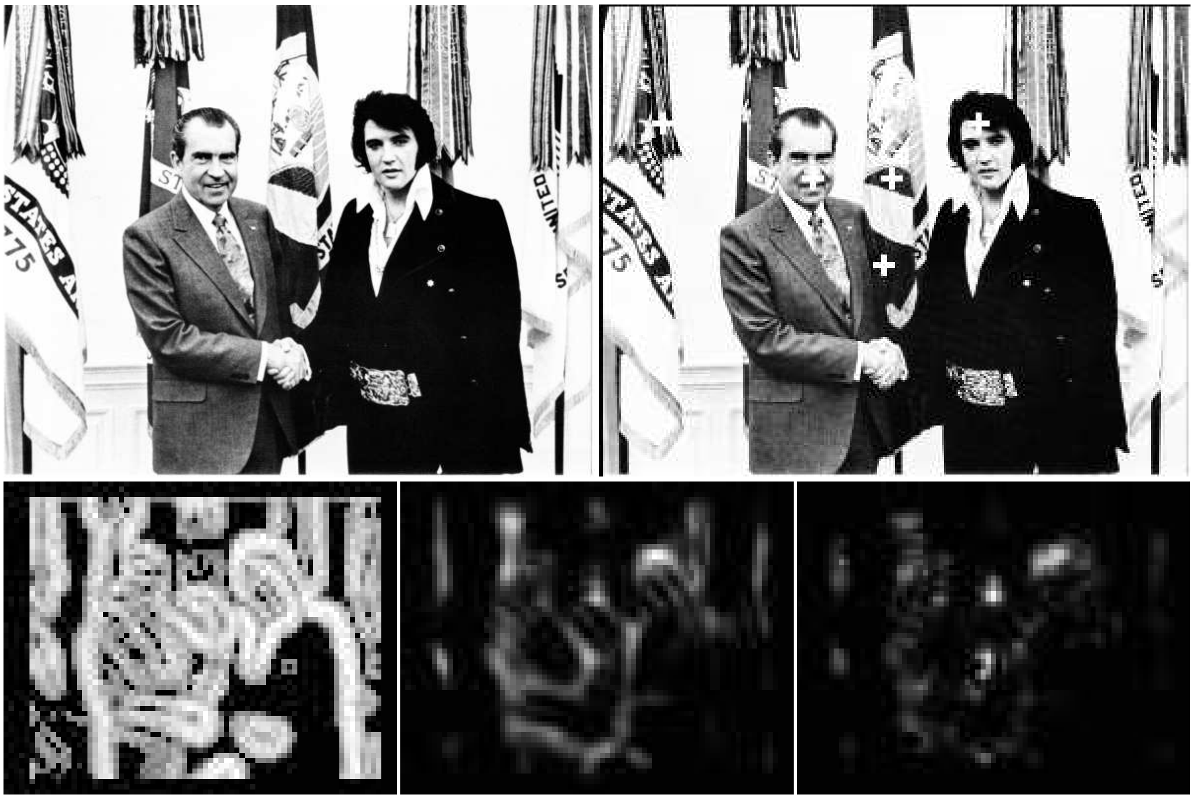
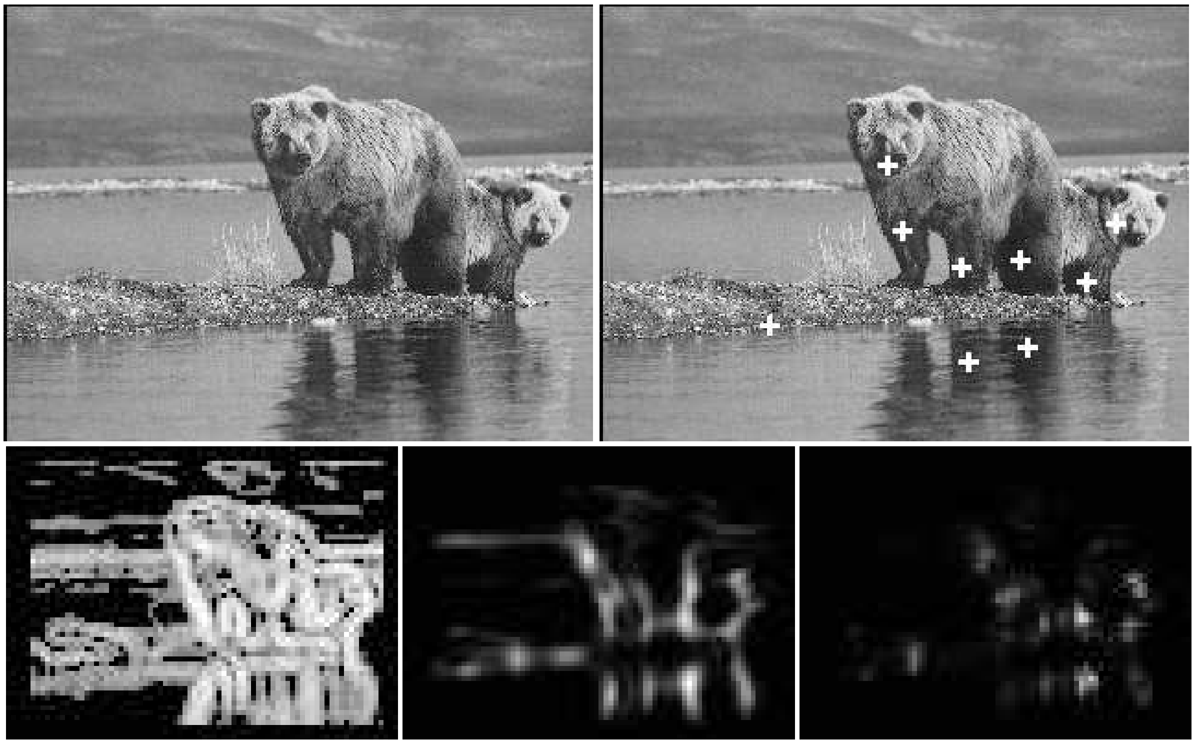


Figure 4:

Figure 5:



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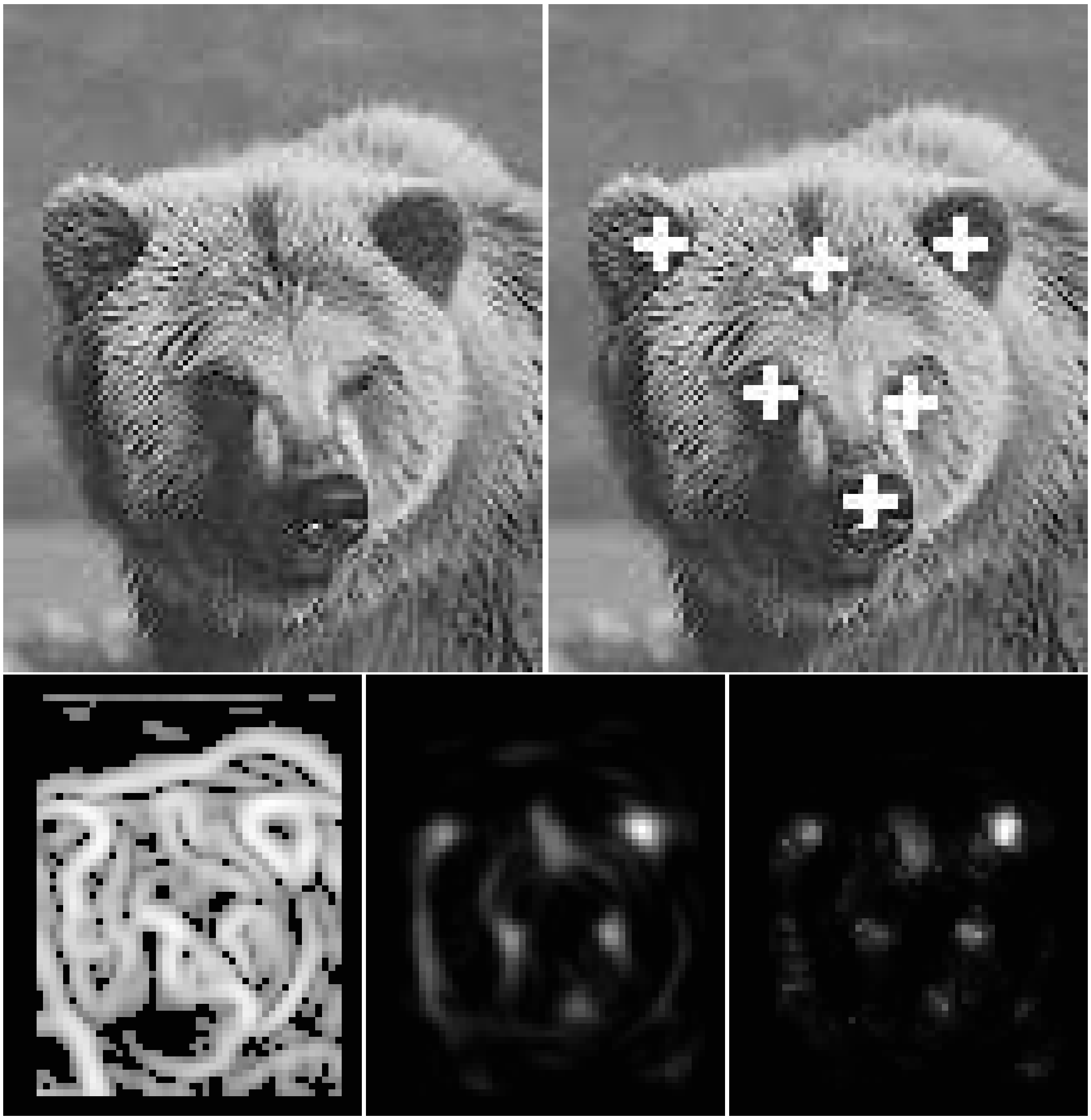

Figure 7:

Figure 8:

Figure 9:

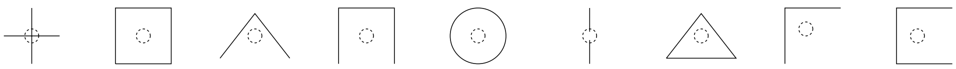

Figure 10:

Figure 11: