CS 6550 Algorithms (Fall '10)

Instructor: Asaf Shapira

Home Assignment 1

Due date: 09/23/10

Please submit organized and well written solutions!

Problem 1. Show that if we randomly throw n balls into n bins by choosing for each ball d possible bins and placing the ball in the least loaded one, then the maximum load is whp at most $\ln\ln n/\ln d + O(1)$.

Problem 2. Suppose we modify Papadimitriou's random-walk algorithm for 2-CNF, and instead of starting from an arbitrary assignment, we start the algorithm on a random assignment. What is the expected running time of the new algorithm?

Problem 3. Modify Schöning's random-walk algorithm so that it would be able to decide (with high probability) in time $\text{poly}(n)(3/2)^n$ if a 4-CNF formula is satisfiable.

Problem 4. Consider the following balls-and-bins process; we start by throwing n balls into n bins. From that point on, at each iteration we remove every bin that is occupied by the balls thrown at the previous iteration, and then throw n "new" balls into the remaining bins. The process ends when there are no more bins left. Show that the expected number of iterations performed by the process is $O(\log^* n)$.

Hint: Break the process into several "sub-steps" as in the coupon collector problem.

Problem 5. A 2-coloring of a graph is called Δ-good if it does not create monochromatic triangles[1]. Consider the following algorithm for finding a Δ-good coloring of a graph $G = (V, E)$; Starting from an arbitrary 2-coloring of $V(G)$, while there are monochromatic triangles, pick one such triangle Δ, randomly pick one of the 3 vertices of Δ and change the color of that vertex.

Show that if G is 3-colorable, then with high probability, this algorithm will find a Δ-good coloring of G after a polynomial number of recoloring iterations.

Hint: Start by convincing yourself that a 3-colorable graph indeed has a Δ-good coloring. Also, since the problem of finding a Δ-good coloring in *arbitrary* graphs is NP-hard, you will have to "use" the assumption that the input is 3-colorable.

[1] A triangle in G is monochromatic if its three vertices are colored by the same color.