Problem 1. Suppose $\mathcal{P} = \{V_1, \ldots, V_k, U_1, \ldots, U_t\}$ is a partition of $V(G)$ with $|V_1| = \cdots = |V_k|$ and $\sum_{i=1}^t |U_i| \leq \epsilon n$. Show how to turn $\mathcal{P}$ into an equipartition $\mathcal{P}'$ of order $k$ satisfying $q(\mathcal{P}') \geq q(\mathcal{P}) - 8\epsilon$.

Problem 2. Show that the statement of the regularity lemma remains valid even if instead of asking for an equipartition $\{V_1, \ldots, V_k\}$ in which the number of irregular pairs is bounded by $\epsilon k^2$, we ask that for every $i$ there would be at most $\epsilon k$ indices $j$, for which $(V_i, V_j)$ is irregular.

Hint: Markov’s Inequality.

Problem 3. Suppose $H$ is a 4-uniform hypergraph on $n$ vertices that does not contain 9 vertices spanning more than 2 edges. Show that $H$ has $o(n^2)$ edges.

Hint: Give two proofs, one via the graph removal lemma for $K_4$ and one via the (6,3)-Problem.

Problem 4. Let $T(n)$ denote the number of triangle-free graphs on $n$ (labeled) vertices. Show that $T(n) = 2^{(\frac{1}{4} + o(1))n^2}$.

Problem 5. Show that for every $\epsilon > 0$ there is $n_0 = n_0(\epsilon)$, so that if $G$ is a graph on $n \geq n_0$ vertices and $\delta(G) \geq n/2$ then $G$ contains $(1 - \epsilon)n/4$ vertex-disjoint copies of $C_4$.

Problem 6. Let $E$ be a homogenous linear equation $\sum_{i=1}^k a_i x_i = 0$ (with $k \geq 3$ and $a_i \in \mathbb{Z}$) and denote by $R_E(n)$ the size of the largest subset of $[n]$ containing no solution to $E$ with all $x_i$ being distinct.

- Show that if the coefficients of $E$ satisfy $\sum_{i=1}^k a_i \neq 0$ then $R_E(n) = \Omega(n)$.
  
  Hint: Start by solving the problem when the equation is $x + y = z$.

- Show that if the coefficients of $E$ satisfy $\sum_{i=1}^k a_i = 0$ then $R_E(n) = o(n)$.
  
  Hint: Apply Szemerédi’s Theorem or the removal lemma for digraphs (preferably both).

- Show that if $E$ is of the form $\sum_{i=1}^k a_i x_i = \left(\sum_{i=1}^k a_i\right) x_{k+1}$, with all $a_i > 0$, then we have $R_E(n) \geq n/C^{\sqrt{\log n}}$ for some constant $C$ that may depend on $a_1, \ldots, a_k$. Actually, find a subset of $[n]$ of this size where the only solution to $E$ is when $x_1 = x_2 = \ldots = x_{k+1}$. 