Problem 1. Show that there is no absolute constant $C$, so that for every $\epsilon > 0$ every graph has an $\epsilon$-regular partition of order at most $(1/\epsilon)^C$.

Problem 2. Show that there is a $K_7$-free graph which has independence number $o(n)$ and more than $n^2/3$ edges. Also, show that if $G$ is a $K_7$-free graph with $(1/3 + \epsilon)n^2$ edges then it has independence number at least $\delta(\epsilon)n$.

Problem 3. Let $G$ be a graph with $pn^2/2$ edges. We say that $G$ satisfies property $P$ if it contains at most $(p^4 + o(1))n^4$ copies of $C_4$. We say that $G$ satisfies property $Q$ if all but $o(n^2)$ pairs of vertices $u, v$ satisfy $d(u, v) \leq (p^2 + o(1))n$, where $d(u, v)$ is the co-degree of $u, v$, that is, the number of vertices that are adjacent to both $u$ and $v$. Show that $P$ and $Q$ are equivalent.

Problem 4. The Erdős-Hajnal Theorem states that for every graph $H$ there is a constant $c = c(H)$ so that every $n$-vertex graph that has no induced copy $H$, contains either a clique or an independent set of size $2^c \sqrt{\log n}$. Derive the Induced Ramsey Theorem from the Erdős-Hajnal Theorem.

Hint: Show that if $n \geq n_0(|H|)$ and $G$ is an $n$-vertex graph with no clique or independent set of size $2 \log n$, then in any 2-coloring of $E(G)$ we can find a monochromatic induced copy of $H$.

Problem 5. Let $G$ be a Red/Black-coloring of $K_m$ where the largest Red clique has size $t - 1$. Let us define a 2-coloring of the complete 3-uniform hypergraph on $2^m$ vertices as follows: we think of the vertices as strings in $\{0, 1\}^m$ ordered lexicographically, with entries indexed by the vertices of $G$, and define $\delta(x, y)$ to be the largest entry where two vectors $x, y \in \{0, 1\}^m$ differ. Then we color each triple of vertices $x < y < z$ with the color given to the edge $(\delta(x, y), \delta(y, z))$ in $G$.

- Show that this coloring has a red $K_3^3$.
- Show that there is a function $f$ so that this coloring does not contain a red $K_{f(t)}^3$.
  Hint: Use Ramsey’s Theorem.
- Suppose we change the coloring so that if $(\delta(x, y), \delta(y, z))$ is red then we color $(x, y, z)$ red, but if $(\delta(x, y), \delta(y, z))$ is black then we color $(x, y, z)$ black if $\delta(x, y) < \delta(y, z)$ and grey if $\delta(x, y) > \delta(y, z)$. Deduce from the previous item that there is a function $g(t)$, so that if $G$ does not contain a red $K_t$ or a black $K_{g(t)}$ then this coloring has no Red, Black or Grey $K_3^3$.
- Use the previous item to deduce that $R_3(t, t, t) \geq 2^{R_2(t, g(t))}$, where $R_2(s, t)$ is the usual Ramsey number for graphs.