0366.4118 Percolation on Finite Graphs

Spring Semester 2026

Homework assignment 1

Not for submission/not to be graded

Problem 1. Find a threshold for the appearance of a path of length 2 in $G(n,p)$. [Use either the second moment or multiple exposure for the positive part.]

Problem 2. Let $p = o(1/n)$. Prove that $G \sim G(n,p)$ is acyclic with high probability.

Problem 3. Let $c_1 > 0$ be a constant. Prove that there is a constant $c_2 = c_2(c_1) > 0$ such that $G \sim G\left(n, \frac{c_1}{n}\right)$ with high probability contains an induced path of length at least $c_2 \ln n$. [**Hint:** try repeatedly to construct the required induced path, building it an edge after an edge. You are likely to fail many times, but persistence pays off (here too).]

Problem 4. Let $\varepsilon > 0$, and let L_1 be the largest component of $G \sim G\left(n, \frac{1+\varepsilon}{n}\right)$. Prove that there is $\delta = \delta(\varepsilon) > 0$ such that with high probability

$$|E(L_1)| - |V(L_1)| \geqslant \delta n \,.$$

Problem 5. Prove that $G \sim G\left(n, \frac{c}{n}\right)$ is with high probability planar for $c < 1$, and is with high probability non-planar for $c > 1$.