0366.4118 Percolation on Finite Graphs

Spring Semester 2026

Homework assignment 2

Not for submission/not to be graded

Problem 1. Let $c > 1$ be a constant. Argue that there exists $\delta = \delta(c) > 0$ such that with high probability in $G\left(n, \frac{c}{n}\right)$, there are at least δn vertices in the giant component of the random graph not belonging to any cycle.

Problem 2. Let $0 \leqslant k < \ell$ be integers, and let $c > 1$ be a constant. Prove that a random graph $G \sim G\left(n, \frac{c}{n}\right)$ contains with high probability a cycle C of length $k \pmod{\ell}$.

Problem 3. Show that for every $\varepsilon > 0$ there is $C > 0$ such that a random graph $G \sim G\left(n, \frac{C}{n}\right)$ contains with high probability two vertex disjoint cycles of length at least $\left(\frac{1}{2} - \varepsilon\right) n$ each.

Problem 4. Let $c > 1/2$. Show that with high probability a random graph $G \sim G\left(n, \frac{c \ln n}{n}\right)$ has a unique connected component of size (at least) $n - o(n)$, with all other connected components (if any) being isolated vertices.

Problem 5. Prove that for $p(n) = \frac{\ln n + \omega(n)}{n}$, with $\lim_{n\to\infty} \omega(n) = \infty$, a random graph $G \sim G(n, p)$ contains with high probability a spanning tree with $n - o(n)$ leaves. [**Hint:** you can argue first that such a graph contains whp a forest with $o(n)$ components, no isolated vertices, and $n - o(n)$ leaves.]

Problem 6. Prove that for $p(n) = \frac{\ln n + \ln \ln n + \omega(n)}{n}$, with $\lim_{n\to\infty} \omega(n) = \infty$, and assuming n is even, with high probability every edge of a random graph $G \sim G(n, p)$ participates in a perfect matching.