COMPLEXITY: Exercise No. 2
due April 28

1. Show that a 1-tape DTM that can insert or delete a symbol anywhere in its tape (in a single step) that runs in time $t(n)$, can be simulated by an ordinary 1-tape DTM with time $O(t^2(n))$.

2. Prove that any 1-tape DTM with an infinite tape (i.e., the head of the machine can go any distance to the left or the right on the tape without limitations. Initially, the tape contains the input: a finite sequence of letters without the letter $\sqcup$, and all the other letters on the tape are $\sqcup$. The head is positioned on the first letter to the left of the input) whose time complexity is $t(n)$ can be simulated by an ordinary 1-tape DTM with time complexity $t(n)$ (exactly!)

3. Consider a 1-tape DTM that after each step can move its head either one step right or jump to the first position in the tape (but it can not move one step left). Show that such a machine can simulate an ordinary 1-tape DTM whose time complexity is $t(n)$, in $O(t(n)^2)$ time.

4. Show that any RAM that runs in time $t(n)$ can be simulated by a RAM without indirect addressing in time $O(t^c(n))$ for some constant c. (assume the input to the original RAM is in a single input register. Explain why this assumption is necessary and why it can be assumed without loss of generality).

5. Prove that for any $f(n)$ and $\epsilon > 0$, $\mathbf{SPACE}(f(n)) \subseteq \mathbf{SPACE}(\epsilon f(n))$.

6. a. Prove that if $t(n)$ is a complexity function such that $t(n_0) \leq n_0 + 1$ for some n_0 then any language in $\mathbf{TIME}(t(n))$ is regular (a language is regular if it can be accepted by a finite automaton).
   b. Show a language L in $\mathbf{TIME}(n+2)$ which is not regular.