COMPLEXITY: Exercise No. 4
due May 26

1. True, false or open? explain: $\mathbf{SPACE}(n^3) \subseteq \mathbf{NSPACE}(n)$.

2. Is the following problem in **NL**?
   Given an undirected graph G, does G have exactly two connected components ?

3. Prove that the following problems are **NP**-complete:

   CSP(ONE):
   INSTANCE: A CNF Boolean formula Φ.
   Question: Is there an assignments to the variables of Φ such that in each clause of Φ there is *exactly* one satisfied literal ? (Hint: Use X3C)

   MONOTONE 3-SAT:
   INSTANCE: A CNF Boolean formula Φ in which each clause is either of the form $(x + y + z)$ or of the form $(\overline{x} + \overline{y} + \overline{z})$ (x, y, z are variables, not literals).
   Question: Is there an assignment that satisfies Φ ?

   In an undirected graph $G = (V, E)$, a *vertex cover* is a set $U \subseteq V$, such that for every edge $(u, v) \in E$ either $u \in U$ or $v \in U$ (or both).
   VERTEX COVER (VC):
   INSTANCE: A graph $G = (V, E)$ and an integer k.
   QUESTION: Does G have a vertex cover of size $\leq k$?

   In an undirected graph $G = (V, E)$, a *clique* is a set $U \subseteq V$, such that every two vertices from U are connected by an edge.
   CLIQUE:
   INSTANCE: A graph $G = (V, E)$ and an integer k.
   QUESTION: Does G have a clique of size $\geq k$?

   In an undirected graph $G = (V, E)$, a *dominating set* is a set $U \subseteq V$, such that for every $v \in V \setminus U$ there is $u \in U$ such that $(u, v) \in E$.
   DOMINATING SET (DS):
   INSTANCE: A graph $G = (V, E)$ and an integer k.
   QUESTION: Does G have a dominating set of size $\leq k$?

   1/2-INDEPENDENT SET:
   INSTANCE: A graph $G = (V, E)$.
   QUESTION: Does G have an independent set of size $|V|/2$?