Problem 1. Let $G$ be a connected plane graph. Show that $(G^*)^*$ is isomorphic to $G$.

Problem 2. Suppose that $T$ is a spanning tree of a plane graph $G$. Show that

$$\{e^* : e \in E(G) \setminus E(T)\}$$

is the edge set of a spanning tree of $G^*$.

Problem 3. A plane graph is *face-regular* if all its faces have the same degree.

(a) Characterise all plane graphs that are both regular and face-regular.

(b) Show that exactly five of these graphs are simple and 3-connected. (These are the Platonic graphs – the skeletons of the five Platonic solids.)

Problem 4. Suppose that $G$ is a 2-connected simple plane graph. Prove that $G$ is bipartite if and only if the boundary of every face of $G$ is an even cycle.